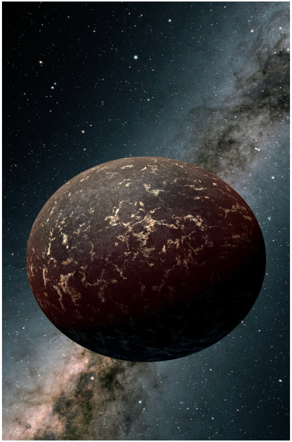
Internal and orbital dynamics of terrestrial planets

Michaela Walterová

February 26, 2021

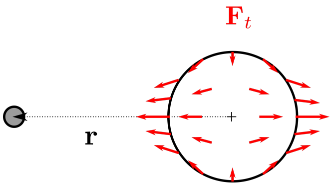
Outline



Credit: NASA/JPL-Caltech/R. Hurt, IPAC

- Tidal interaction
- Parametric studies
- Coupled thermal–orbital evolution (single planet)
- Effect of a second planet
- Numerical model
- Conclusions

Differential gravitational force in an extended body



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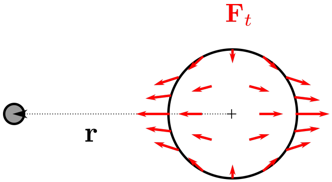
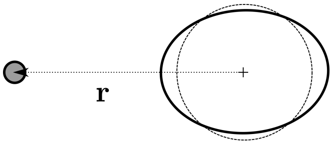
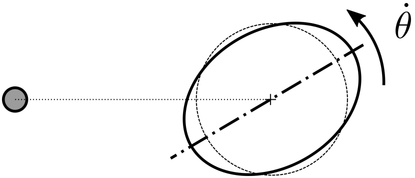


No relative rotation or perfect elasticity:



Energy dissipation in the planet → lagging of the tidal bulges



Tidal interaction

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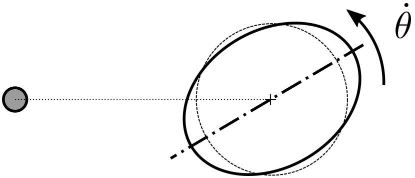

Gravitational action on the bulges:

→ Tidal torque (**spin rate evolution**)
→ Transversal acceleration/decceleration of the perturber (**orbital evolution**)

Tidal interaction

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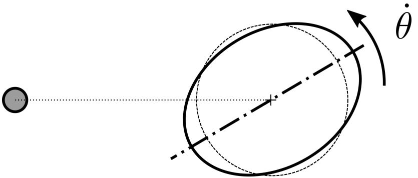


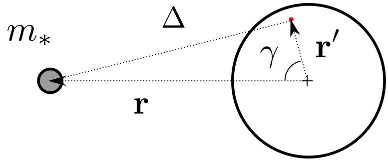
Tidal potential defined as $\mathbf{F}_t = \nabla \Phi_t$

Additional potential induced by tidal distortions

$=$ **Disturbing function** $\mathcal{R}$

→ Lagrange planetary equations (for a, e, i, ω, Ω, M)

Tidal loading of an extended body



$$\mathbf{r}' = (r', \vartheta', \varphi')$$
$$\mathbf{r} = (r, \vartheta, \varphi)$$

Tidal interaction

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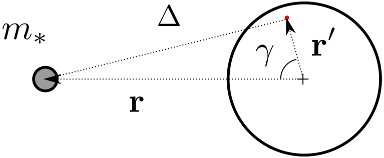

$$\mathbf{r}' = (r', \vartheta', \varphi')$$
$$\mathbf{r} = (r, \vartheta, \varphi)$$

Tidal potential:

$$\Phi_t(r', \vartheta', \varphi') = \frac{\mathcal{G} m_*}{\Delta} = \frac{\mathcal{G} m_*}{r} \sum_{l=2}^{\infty} \left(\frac{r'}{r}\right)^l \mathcal{P}_l(\cos\gamma) =$$

$$= \frac{\mathcal{G} m_*}{r} \sum_{l=2}^{\infty} \left(\frac{r'}{r}\right)^l \sum_{m=0}^{l} \frac{(l-m)!}{(l+m)!} (2 - \delta_{0m}) \mathcal{P}_{lm}(\cos\vartheta') \mathcal{P}_{lm}(\cos\vartheta) \cos[m(\varphi' - \varphi)]$$

Darwin-Kaula expansion

Darwin (1880), Kaula (1961, 1964)

Expansion of tidal potential to **spherical harmonics** and **Fourier modes**: tidal frequencies $\omega_{lmpq} \approx (l-2p+q)n - m\dot{\theta}$

$$\Phi_t(r',\vartheta',\varphi') = \sum_{l=0}^{\infty}\sum_{m=0}^{l}\sum_{p=0}^{l}\sum_{q=-\infty}^{\infty}\Phi_{lmpq}(r',\vartheta',\varphi')\ ,$$

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where

$$\Phi_{lmpq}(r', \vartheta', \varphi') = \frac{\mathcal{G} m_*}{a} \left(\frac{r'}{a} \right)^l \frac{(l-m)!}{(l+m)!} (2 - \delta_{0m}) \, \mathcal{G}_{lpq}(e) \, \mathcal{F}_{lmp}(i) \, \mathcal{P}_{lm}(\cos\vartheta') \times$$

$$\times \begin{Bmatrix} \cos \\ \sin \end{Bmatrix}_{l-m \text{ odd}}^{l-m \text{ even}} [\nu_{lmpq} - m(\varphi' + \theta)]$$

and $\nu_{lmpq} = (l-2p)\omega + (l-2p+q)M + m\Omega$

Darwin-Kaula expansion

Expansion of the additional potential (disturbing function):

$$\mathcal{R} = \sum_{l=0}^{\infty} \sum_{m=0}^{l} \sum_{p=0}^{l} \sum_{q=-\infty}^{\infty} \mathcal{R}_{lmpq}, \quad \text{where} \quad \mathcal{R}_{lmpq}(\mathbf{r}) = \left(\frac{R}{r}\right)^{l+1} k_l \left[\Phi_{lmpq}(R)\right]_{\text{lag}} ,$$

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or

$$\mathcal{R}_{lmpq} = \frac{\mathcal{G}m_*}{a} \sum_{p'=0}^{l} \sum_{q'=-\infty}^{\infty} \kappa_m \frac{(l-m)!}{(l+m)!} \frac{R^{2l+1}}{a^l \tilde{a}^{l+1}} \mathcal{F}_{lmp}(\tilde{i})\, \mathcal{F}_{lmp'}(i)\, \mathcal{G}_{lpq}(\tilde{e}_1)\, \mathcal{G}_{lp'q'}(e_1) \times$$

$$\times\ k_l \cos[\tilde{\nu}_{lmpq} - \varepsilon_{lmpq} - m\tilde{\theta} - (\nu_{lmp'q'} - m\theta)]$$

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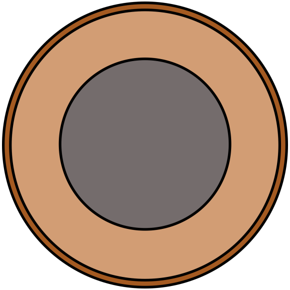
Tidal **Love number** k_l and **phase lag** $\varepsilon_{lmpq} \leftarrow$ interior structure



Complex Love numbers

$$\bar{k}_l(\omega_{lmpq}) = k_l \, \exp\{-\mathrm{i}\varepsilon_{lmpq}\}$$

Layered model:

- liquid core
- viscoelastic mantle
- elastic lithosphere

Normal mode theory:

e.g., Takeuchi and Saito (1972), Sabadini and Vermeersen (2004)

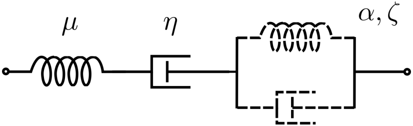
Spheroidal deformations, tractions, and perturbed potential in each layer j given analytically and parameterised by six constants $\bar{C}_i^{(j)}$

$$\rightarrow \bar{k}_l(\omega) = -\bar{C}_3^{(N)} R^l - \bar{C}_6^{(N)} R^{-l-1} - 1$$

Constitutive relation $2\bar{\boldsymbol{\epsilon}}^{\mathrm{D}}(\omega) = \bar{J}(\omega)\bar{\boldsymbol{\sigma}}^{\mathrm{D}}(\omega)$

Andrade rheology: Andrade (1910), Castillo-Rogez et al. (2011)

elastic deformation + viscous creep + anelasticity



$$\bar{J}(\omega) = \frac{1}{\mu} - \frac{\mathrm{i}}{\eta\omega} + \frac{\mu^{\alpha-1}}{(\mathrm{i}\,\zeta\eta\omega)^{\alpha}}\,\Gamma(1+\alpha)$$

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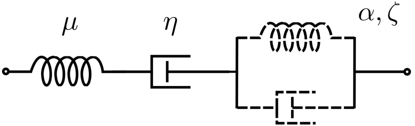

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Interior properties → $\bar{k}_l(\omega_{lmpq})$ → Tidal evolution

Tidal effects

Secular tidal heating   Efroimsky and Makarov (2014)

$$\bar{P}^{\text{tide}} = -\frac{\mathcal{G} m_*^2}{a} \sum_{lmpq} \left(\frac{R}{a}\right)^{2l+1} (2-\delta_{m0}) \frac{(l-m)!}{(l+m)!} \left[\mathcal{G}_{lpq}(e)\right]^2 \left[\mathcal{F}_{lmp}(\beta)\right]^2 \times$$

$$\times\, \omega_{lmpq}\, \mathrm{Im}\left\{\bar{k}_l(\omega_{lmpq})\right\}$$

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+ spin rate evolution

$$C\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = \mathcal{T}\ ,$$

$$\mathcal{T} = -\frac{\mathcal{G}m_*^2}{a}\sum_{lmpq}\left(\frac{R}{a}\right)^{2l+1}\frac{(l-m)!}{(l+m)!}\,m\,\kappa_m\left[\mathcal{G}_{lpq}(e)\right]^2\left[\mathcal{F}_{lmp}(\beta)\right]^2\mathrm{Im}\left\{\bar{k}_l(\omega_{lmpq})\right\}$$

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+ secular orbital evolution (Lagrange planetary equations)
with disturbing function $\mathcal{R} = \mathcal{R}(a, e, i, \omega, \Omega, M; \bar{k}_l)$

Which quantities determine the magnitude of the tidal effects?

- orbital elements (semi–major axis, eccentricity)
- spin rate and obliquity
- perturber mass, planet radius
- interior structure and rheological parameters

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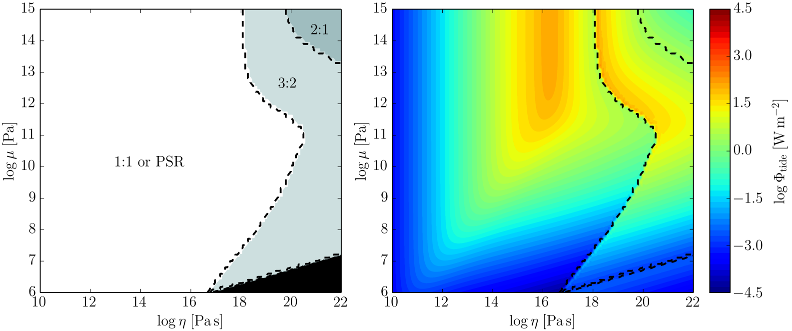
Parametric study for a generic planet

Earth–sized model planet governed by the Andrade rheology

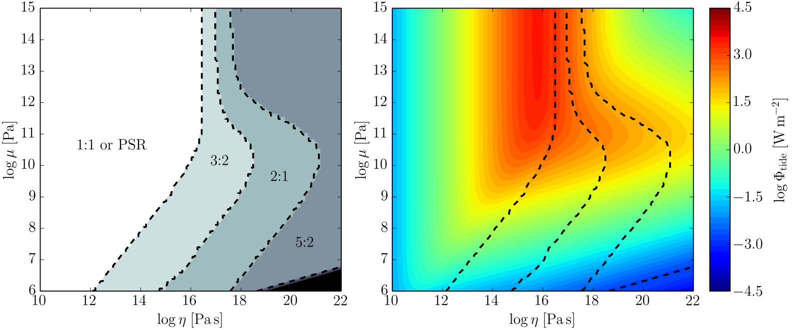
Parameter	Definition	Value	Unit
m_*	Mass of the host star	0.1	$m_\odot$
a	Semi–major axis	0.04	AU
e	Eccentricity	0.0 to 0.5	—
d_{lid}	Lithosphere thickness	50	km
CMF	Core mass fraction	0.3	—
R	Outer radius of the planet	1	$R_\oplus$
α	Parameter of the Andrade model	0.3	—
ζ	Parameter of the Andrade model	1	—

Parametric study for a generic planet

Left: The first/highest stable spin–state encountered during despinning
Right: Corresponding mean surface tidal heat flux $e = 0.05$



Left: The first/highest stable spin–state encountered during despinning
Right: Corresponding mean surface tidal heat flux $e = 0.2$



Left: The first/highest stable spin–state encountered during despinning
Right: Corresponding mean surface tidal heat flux $\mu = 200\,\mathrm{GPa}$

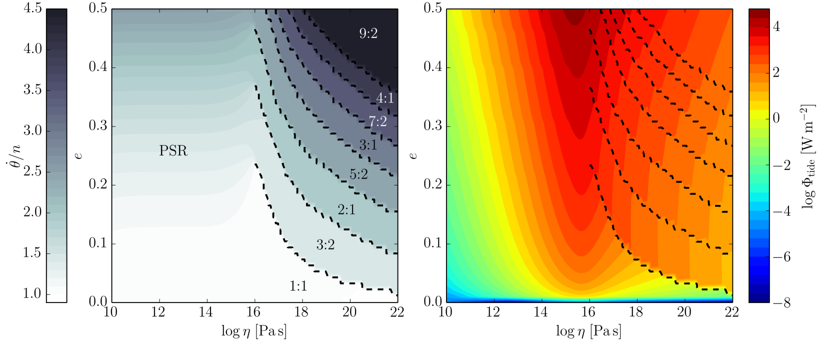
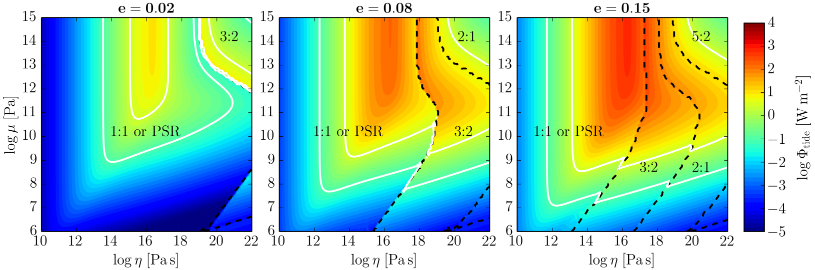


Illustration: Model inspired by Proxima Centauri b

Parameter	Value
$m_* \; [m_\odot]$	0.12
a [AU]	0.0485
$m_\mathrm{p} \; [m_\oplus]$	1.27
$R \; [R_\oplus]$	1.074

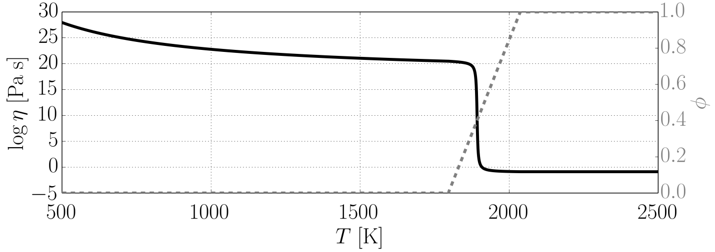


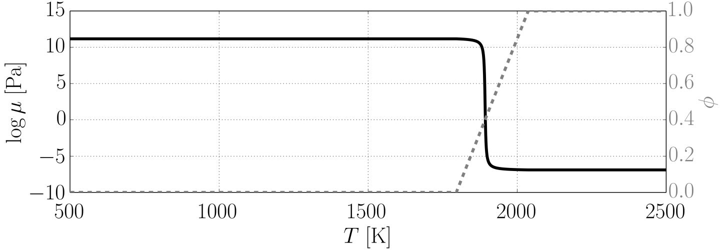
Thermal evolution

What if $\eta = \eta(T)$ and $\mu = \mu(T)$?

Thermal evolution

What if $\eta = \eta(T)$ and $\mu = \mu(T)$? plotted at $p_0 = 1\,\mathrm{GPa}$





Parameterised mantle convection in the **stagnant lid regime**

e.g., Breuer and Spohn (2006), Tosi et al. (2017)

$$\rho_\mathrm{m} c_\mathrm{m} V_\mathrm{m} (1 + St) \frac{\mathrm{d}T_\mathrm{m}}{\mathrm{d}t} = -q_\mathrm{m} A_\mathrm{m} + q_\mathrm{c} A_\mathrm{c} + \bar{P}^\mathrm{tide} \,,$$

$$\rho_\mathrm{c} c_\mathrm{c} V_\mathrm{c} \frac{\mathrm{d}T_\mathrm{c}}{\mathrm{d}t} = -q_\mathrm{c} A_\mathrm{c}$$

Fluxes q_m and q_c depend on the average mantle viscosity η_m

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Temperature–dependent viscosity: Arrhenius law

$$\eta(T) = \eta_0 \exp\left(\frac{\mathcal{E}^*}{R_\mathrm{gas}} \frac{T_0 - T}{T_0 T} \right)$$

with reference viscosity $\eta_0 = \eta(T = 1600\,\mathrm{K})$ + melting

Adiabatic temperature profile in the convecting mantle

$\rightarrow T(r)$, $\eta(r)$, $\mu(r)$, $\phi(r)$
$\rightarrow$ average η_m, μ_m

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At $\phi(r) > 40\%$: formation of a liquid layer – "**magma ocean**"

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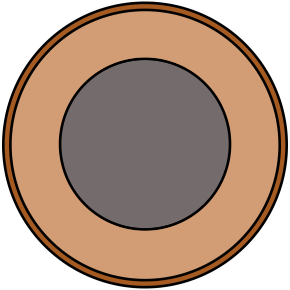

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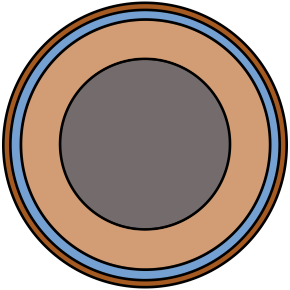
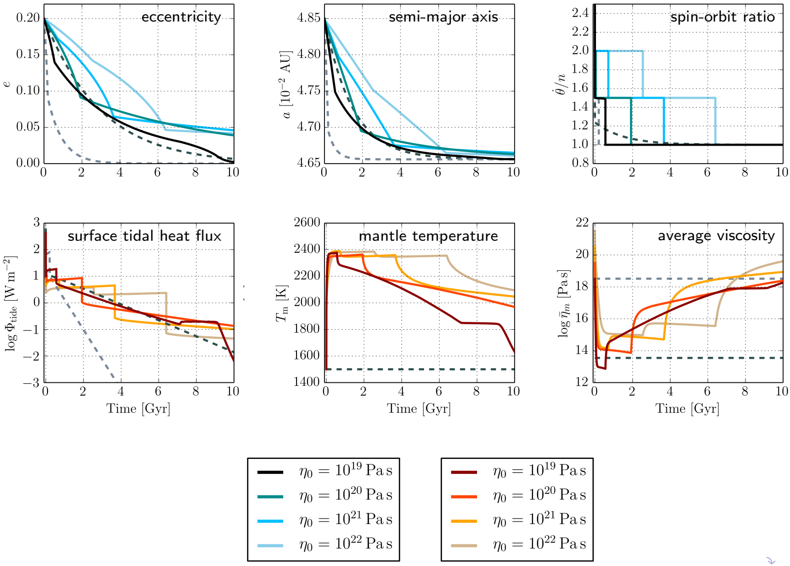


Thermal-orbital evolution



Effect of a second planet

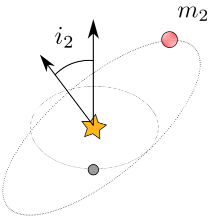
Addition of a **second planet** to the system

$\rightarrow$ third-body perturbations

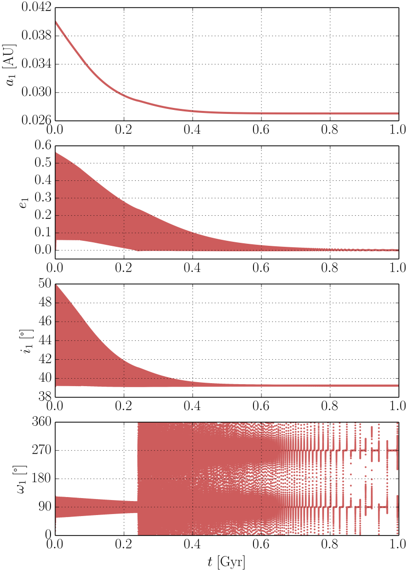
Effect of a second planet

Addition of a **second planet** to the system
→ third-body perturbations

Illustration:
Highly inclined perturber



$m_2 = 1\,m_J,\ a_2 = 1\,\mathrm{AU},\ i_2 = 50^\circ$



Effect of a second planet

Planet–planet tidal potential

$$\mathcal{U}_{lm} = \sum_{n=0}^{\infty} \sum_{p_1=0}^{m} \sum_{p_2=0}^{m-p_1} \sum_{p_3=0}^{p_1} \sum_{p_4=0}^{n} \sum_{q_1=-\infty}^{\infty} \sum_{q_2=-\infty}^{\infty} \frac{1}{a_>^{l+1}} \left(\frac{a_<}{a_>}\right)^{\varrho} \mathcal{F}^0_{lmn\bar{p}} \, \mathcal{G}^A_{lmn\bar{p}\bar{q}}(e_A) \, \mathcal{G}^B_{lmn\bar{p}\bar{q}}(e_B) \times$$

$$\times \left[\begin{Bmatrix} \mathcal{A}_{lm} \\ -\mathcal{B}_{lm} \end{Bmatrix}_{l-m \text{ odd}}^{l-m \text{ even}} \cos \Big[(m - p_1 - 2p_3 - n + 2p_4 + q_1) M_A + (p_1 - 2p_2 + n - 2p_4 + q_2) M_B + \right.$$

$$+ (p_1 - 2p_2 + n - 2p_4)\varpi + m(\alpha - \theta) \Big] +$$

$$+ \begin{Bmatrix} \mathcal{B}_{lm} \\ \mathcal{A}_{lm} \end{Bmatrix}_{l-m \text{ odd}}^{l-m \text{ even}} \sin \Big[(m - p_1 - 2p_3 - n + 2p_4 + q_1) M_A + (p_1 - 2p_2 + n - 2p_4 + q_2) M_B +$$

$$\left. + (p_1 - 2p_2 + n - 2p_4)\varpi + m(\alpha - \theta) \Big] \right]$$

Numerical model

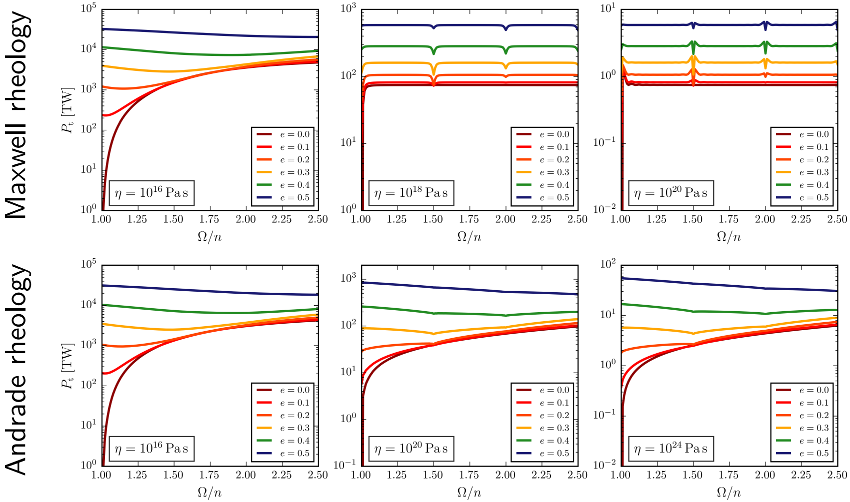
Alternative to the semi-analytical model described earlier

Deformation and dissipation in the planet calculated by numerical tool *Andy4* (prof. Čadek)

details in Tobie et al. (2008), Běhounková et al. (2015)

Time domain, SH decomposition in the lateral direction, FDM in the radial direction

Numerical model - tidal heating



Conclusions

Andrade rheology: complex dependence of **tidal locking** on the rheological parameters (η, μ)

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— Driven by transitions between the SOR (or by the eccentricity)

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Thank you!