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Contents

1 Abstrakt / Abstract

Náplní této práce je analyzovat spektrální metody používané pro určení zdrojových parametrů zemětřesení jako je seismický moment M_0 a rohová frekvence f_c a jejich aplikace na data ze západočeské seismické oblasti. Uvážením dalších předpokladů o zdroji lze dále odhadnout některé důležité parametry jako je poloměr zdroje r nebo pokles napětí ve zdroji $\Delta\sigma$. Určení parametrů je prováděno ve spektrální oblasti pomocí porovnání jednoduchého Bruneho modelu zdroje (nekauzální skluz na kruhové trhlině; spektrální spád ω^{-2}) se záznamem posunutí přímé P nebo S vlny. Metody byly aplikovány na 56 vybraných rojových zemětřesení z oblasti západních Čech z let 2000 a 2008 absolutním a relativním přístupem v několika modifikacích. Absolutní metoda kromě parametrů zdroje umožňuje získat také faktor kvality Q (útlum vlivem prostředí), který zásadním způsobem ovlivňuje určení f_c. Proto byla absolutní metoda aplikována také v modifikaci hromadné inverze, kdy je Q stabilizováno. Ukázalo se, že relativní metoda aplikovaná pomocí poměrů spekter trpí větší nestabilitou a nepřesností řešení. Výsledky obou metod nepotvrdily obecně předpokládanou platnost vzájemné podobnosti slabých a silných zemětřesení, pro kterou je uvažován konstantní pokles napětí. Neurčitost výsledků použitých metod a předpokladů jednoduchosti zdroje ovšem umožňuje pouze hrubý odhad parametrů zdroje.

The aim of this thesis is to analyze the spectral methods used for the determination of the earthquake source parameters like the seismic moment M_0 and the corner frequency f_c and to apply these methods to seismic data from the West Bohemian region. Considering some assumptions about the source the other important parameters like the source radius r or the stress drop in the source $\Delta\sigma$ can be evaluated. Determination of the parameters is performed in the spectral domain by comparing a simple Brune's source model (non-causal slip on a circular rupture; spectral slope ω^{-2}) with the displacement of the P or S wave. The methods were applied to 56 selected earthquakes of the West Bohemian swarms from 2000 and 2008 by the absolute and relative approach in several modifications. The absolute method allows to determine not only the source parameters but also the quality factor Q (attenuation), which significantly affects the determination of f_c. Therefore, the absolute method was applied also as the joint inversion when Q is stabilized. It turned out that the relative method applied by using spectral ratios suffer from greater instability and uncertainty of the solution. The results of both methods did not confirm the generally expected self-similarity of weak and strong earthquakes for which the constant stress drop is considered. However, uncertainty of the results of the used methods and assumptions on the source simplicity allows only a rough estimate of the source parameters.

2 Introduction

One of the main reasons why to study the source parameters of small earthquakes is the chance that understanding of the source processes of the more frequent small earthquakes could help to understand the behavior of the big ones. The assumption of self-similarity between the weak and strong earthquakes was first formulated by Aki (1967) and became a popular and discussed topic for the following decades. According to my knowledge this assumption was not reliably proved neither denied. As many other effects influence the observed data the clear answer is hard to provide.

Source parameters of the small earthquakes are of interest mainly because of the assumption of self-similarity of weak and strong earthquakes. Therefore it is believed that understanding of the source processes retrieved from the more frequent small earthquakes could help to understand the behavior of the big ones. The validity of the self-similarity is widely tried to be verified using the simple source models, usually circular rupture after Brune (1970), with some other assumptions about the source dynamics (e.g. Madariaga, 1976). This simple approaches are used also due to unknown material properties along the raypath which do not allow to apply the complex source model simply. There is also many presumptions about the source which may not be valid for all events but the simple source model can provide the rough estimates of the source parameters at least.

3 Theory

Displacement in the far field in the homogeneous media due to a point source dislocation of strength $M_0(t)$ is

$$\mathbf{u}(\mathrm{x}, t) = \frac{1}{4\pi\varrho\alpha^3}\Re_\alpha \frac{1}{R}\dot{M}_0\left(t - \frac{R}{\alpha}\right) + \frac{1}{4\pi\varrho\beta^3}\Re_\beta \frac{1}{R}\dot{M}_0\left(t - \frac{R}{\beta}\right) \tag{1}$$

(Aki and Richards, 2002, eq. 4.32) where α and β are velocities of seismic waves and $\Re_\alpha$ and $\Re_\beta$ are the corresponding radiation pattern (RP) corrections. Quantity ρ is density and R is the hypocentral distance. From (1) we can formalize the relation for seismic moment

$$M_0 = \frac{4\pi\rho c^3 R\Omega_0}{\Re_c F_c^{surf}} \tag{2}$$

where Ω_0 is the amplitude low-frequency spectral level (or plateau; $\Omega_0 \sim \Omega(f \to 0)$), c is the velocity of seismic wave (α or β), $\Re_c$ is corresponding radiation pattern correction and F_c^{surf} is corresponding free surface correction. Calculation of the seismic moment is independent of the source model.

3.1 Source model

Moving to the source itself, in the thesis I described the sources localized within the WEBNET seismic network using the Brune type source model. The spectral representation of the displacement in the source has form

$$|U(f)| = \frac{\Omega_0}{\left[1 + \left(\frac{f}{f_c}\right)^{\gamma n}\right]^{1/\gamma}} \tag{3}$$

(Brune, 1970) where f_c is the corner frequency of the earthquake, $\gamma = 1$ and n is the high-frequency spectral falloff (for Brune model $n = 2$). Therefore for such simple model of the source we are able to obtain two independent quantities – Ω_0 and f_c. Assuming an instantaneous shear stress release on a circular dislocation (Snoke, 1987) the radius of such circular fault is

$$r = \frac{k_c \beta}{f_c} \tag{4}$$

where k_c is the model dependent constant. In case of Brune model $k_c = 0.37$. This model has been heavily used and published in numerous studies. Further, quasidynamic circular model was developed by Madariaga (1976) who studied a plane circular faulting with fixed rupture velocity. The model is quasidynamic since the effective stress on the fault is specified (Gibowicz and Kijko, 1994). For this model k_c is a function of azimuth (i.e. angle between the fault and ray direction) but in case of sufficiently dense network coverage one can apply the average values of the coefficients. Assuming the rupture velocity v_r = 0.9β ($\beta = 3.5$ km/s), then the average values are $k_\alpha = 0.32$, $k_\beta = 0.21$.

3.2 Stress drop

For a finite fault we define the static stress drop as stress drop integrated over the fault area normalized by the fault area. Approximating the fault area by a square with side of length $\widetilde{L}$ (characteristic dimension) with an average displacement $\bar{D}$ the static stress drop is related to the strain change $\bar{D}/\tilde{L}$ and according to Hook's law

$$\Delta\sigma = C\mu\frac{\bar{D}}{\widetilde{L}} \tag{5}$$

where C is non-dimensional constant that depends on the fault geometry (Lay and Wallace, 1995). For a circular shape with radius r the constant $C = {}^{7\pi}/_{16}$. By substitution of μ from (6) we get

$$\Delta\sigma = \frac{7}{16}\frac{M_0}{r^3} \tag{6}$$

which is relation obtained by Eshelby (1957). Looking at (6), the possible error in r (i.e. f_c) is transferred into the error of stress drop with the third power. This is why one should be cautious when interpreting the results because increase of the corner frequency f_c only by 20 % will double the stress drop $\Delta\sigma$. Relation (6) was derived for a circular rupture buried in a homogeneous infinite material which is not valid in the vicinity of the earthquake asperity and also the Hook's law in not valid here and this brings another uncertainty to $\Delta\sigma$. A number of modeling assumptions are made but they are still very likely different from the real physics of the rupture processes; e.g. variations in rupture speed will cause a change in corner frequency even if the stress drop remains constant (Shearer, 2009). Therefore values of the stress drop are only rough estimates and credibility of these values should be considered to be accurate in terms of orders.

3.3 Apparent stress

Another quantity characterizing the earthquake process is the apparent stress which is a ratio of radiated energy E_c to seismic moment M_0

$$\sigma_a = \mu\frac{E_c}{M_0} \tag{7}$$

(Boatwright, 1984), where index c is related to the specific type of wave (either α or β). This relation expresses the efficiency of the energy radiation. First was assumed that the apparent stress should be constant for all earthquakes but many studies show (e.g. Aki, 1967; Kanamori and Anderson, 1975; Mayeda et al., 2005) that the self-similarity may not be valid for all. But it is still question whether the non-self-similarity is real or comes from uncertainties in estimation of the E_c, M_0 parameters, predominantly the first one.

Corner frequency - Snoke's approach

As a second approach to retrieve the corner frequency I applied the method of Andrews (1986) and Snoke (1987, eq. 3), which uses the integral of the square of the ground velocity spectrum J (after the attenuation correction; including the correction for limited bandwidth) for direct determination of f_c from the spectrum without the need of inversion:

$$f_c(J) = \left(\frac{J}{2\pi^3\Omega_0^2}\right)^{\frac{1}{3}} \tag{8}$$

where

$$J = 2\int_0^\infty |\dot{u}(f)|^2 df \tag{9}$$

This kind of derivation of corner frequency can be easily automated if J and Ω_0 are estimated correctly.

4 Application

I used the Brune source model in frequency domain

$$\Omega(f) = \frac{\Omega_0 e^{-\pi f t/Q}}{1 + \left(\frac{f}{f_c}\right)^n} \tag{10}$$

(Brune, 1970) with the attenuation term $e^{-\pi f t/Q}$, where Ω_0 is the low frequency spectral level proportional to the seismic moment M_0, t is the wave travel time, Q is the quality factor for the whole ray path, f_c is the corner frequency and n = 2. The exponential term is responsible for the anelastic attenuation along the ray path. Because I assume a point source (i.e. no effects of source directivity), f_c is assumed to be the same for all stations. The spectrum of the P or S wave displacement was calculated using the multitaper approach (MTT) which smooth the spectrum.

A time window of 1 sec duration was used to calculate the spectrum of P wave and 1.5 sec for S wave. It contains 0.5 sec of noise before the arrival time and the remaining part includes the signal of the wave with almost 100 % of its energy. The noise spectrum was calculated from the first half of the time window and replicated while the signal spectrum was calculated from the whole window using the MTT. Spectrum was interpolated in order to obtain equidistant frequency spacing in the logarithmic scale (12 points per decade). I did not apply any smoothing operator because the MTT smooth the spectrum itself. I applied both absolute and relative spectral methods to retrieve the source parameters.

4.1 Absolute methods

In the absolute methods the seismic moment M_0 and the corner frequency f_c are searched for as the source parameters. The quality factor Q for the whole ray path is an additional but crucial parameter which substantially influences the resulting source parameters.

4.1.1 Individual event inversion

In the individual event inversion the spectrum of the P or S wave displacement is compared to the spectral source model and minimum of the residual function

$$SR = \sum_{i=1}^{N} || \log \Omega_{model}(f_i) - \log \Omega_{data}(f_i)||_{L_2} \tag{11}$$

where N is the number of discrete frequencies at which the model and data are compared, is searched for using the *Nelder-Mead simplex method* (implemented in MATLAB). The simplex method always converged to the minimum of the residual function when the initial values of fitted parameters were in reasonable intervals. For f_c the initial value should be in the range of the observed data, i.e. 1-100 Hz and for Q the initial value of 200 was used (Stein and Wysession, 2003), because all the stations are surface stations built on a hard rock (crystalline or metamorphic units). Different initial values of fitted parameters did not affect the final solution. To stabilize the inversion I made an assumption of a common f_c for all stations for the analyzed event which can be valid in the first approximation of the source only. The assumption of a common f_c can be used only if one does not expect any directivity effects of the source. The residual function in such case is modified to

$$SR_{all} = \frac{1}{N} \sum_{i=1}^{N} SR_i \tag{12}$$

where N is the number of stations where the event was analyzed.

4.1.2 Joint inversion

For some selected events I performed a joint inversion of corner frequency f_c and quality factor Q over multiple events and all suitable stations. The source model was assumed to be the same as in previous section, i.e. the same f_c over all stations for each particular event. Second constraint was related to the quality factor Q, i.e. the Q was the same for each particular station. This constraint I could use because of similar ray paths from the closely collocated events. The residual function is then defined as

$$SR_{joint} = \sum_{i=1}^{N*M} || \log \Omega_{model}(f_i) - \log \Omega_{data}(f_i)||_{L_1} \tag{13}$$

where N is the number of stations and M is the number of events; I searched for N unknown Q and M unknown f_c. The low-frequency spectral level was fixed to the same values as in the previous absolute approach and therefore the seismic moment M_0 was not affected by the inversion.

4.2 Relative methods

The relative empirical Green's function method (EGF; or spectral ratio method) is well known and widely used for analysis of collocated events in variable environments. In this method the weaker event is assumed to be a delta function $\delta(t)$ with respect to the stronger event. Aim of the EGF method is to separate the source time function from the stronger event by deconvolution of the weaker event. In an ideal case, the division of the spectra will remove all effects except for the source properties. The main advantage of this method is that almost all path and site attenuation effects are eliminated by the spectral division. The spectral ratio of observed data is

$$\Psi^{obs}(f) = \frac{U_1(f)}{U_2(f)} = \frac{\Omega_1(f)}{\Omega_2(f)} \tag{14}$$

where $U_1(f)$ and $U_2(f)$ are the displacement spectra of the stronger and weaker events, respectively. The $\Omega_2(f)$ term is the spectrum of the weaker event which has nearly flat amplitude up to its corner frequency f_{c2} and is considered to be similar to a spectrum of the $\delta(t)$ pulse. The spectral ratio of the model using (10) is defined as

$$\Psi^{mod}(f) = \Omega_{0r} \frac{1 + \left(\frac{f}{f_{c2}}\right)^2}{1 + \left(\frac{f}{f_{c1}}\right)^2} \tag{15}$$

(e.g. Viegas et al., 2010) where $\Omega_{0r} = \Omega_{01}/\Omega_{02}$ is the low-frequency spectral ratio of the stronger and weaker event and f_{c1} and f_{c2} are their corner frequencies, respectively. The best fit between the observed data ratios and the model is solved as an inverse problem by minimizing the differences in L_2 norm. In this analysis the residual function is optimized in the form

$$SR_{EGF} = \sum_{i=1}^{N*M} ||\Psi_i^{obs}(f) - \Psi_i^{mod}(f)||_{L_2} \tag{16}$$

(similar to e.g. Kwiatek et al., 2011) where N is the number of event pairs and M is the number of stations. The analysis can be performed for

(a) one pair of events ($N = 1$) at M stations (i.e. I invert for one f_{c1} and one f_{c2} at all stations); or

(b) N event pairs and M stations simultaneously (i.e. I invert for one f_{c1} and N values of f_{c2} for each pair of events).

In addition, case (b) can be turned into (a) by creating one average EGF from more weak events of similar magnitude by stacking of their spectra (Shearer et al., 2006).

4.3 Data

In my thesis I employed the high-quality seismic data recorded by the WEBNET seismic network (Fischer et al., 2010) (Fig. 1) to bring new insights into understanding of the source processes in this unique area of intracontinental seismicity.

The absolute methods were applied to 56 swarm-events from the West Bohemia region in years 2000 and 2008 (Fig. 2). The first results of the absolute approach applied to the P waves were published in Michálek and Fischer (2013). In the thesis the results were examined in more details extending the error analysis and broadened in terms of new methods and by additional analysis of S waves (applied to 24 events from 2008 swarm).

The relative methods were applied to a small group of six events from the 2008-earthquake-swarm. The selection of these events was based on results from the previous cluster analysis (Fischer and Michálek, 2008) where the similarity of the P-wave seismograms was tested by the cross-correlation analysis. The aim of application of the relative methods was not to process a large number of events but to test the stability of the methods and their uncertainties.

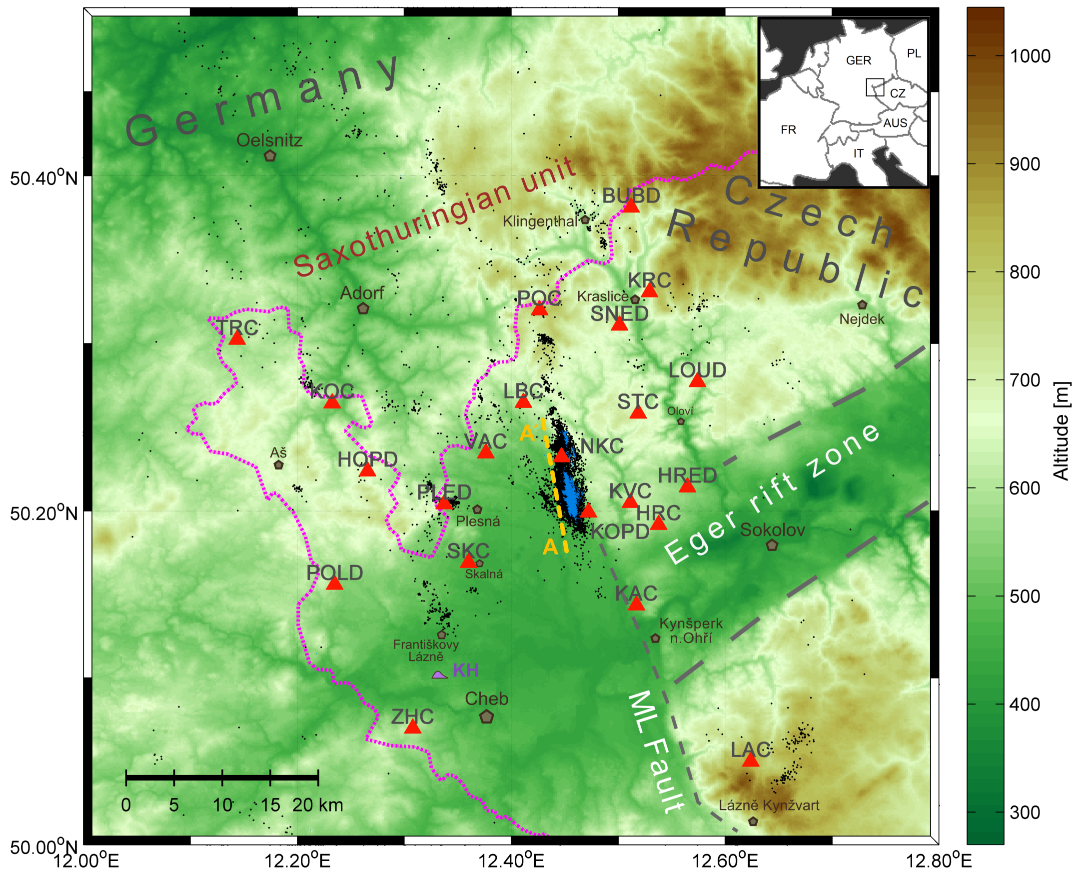

Figure 1: Map of WEBNET network and epicenters from 1991 to 2010 (black dots) in the West Bohemia/Vogtland region. The earthquake swarms 2000 and 2008 are highlighted in blue color. The red triangles are WEBNET stations. The depression to the south from the NKC station is the tertiary Cheb basin; its eastern edge is terminated by the intersection with the Eger rift. (topography based on SRTM3-Version 2)

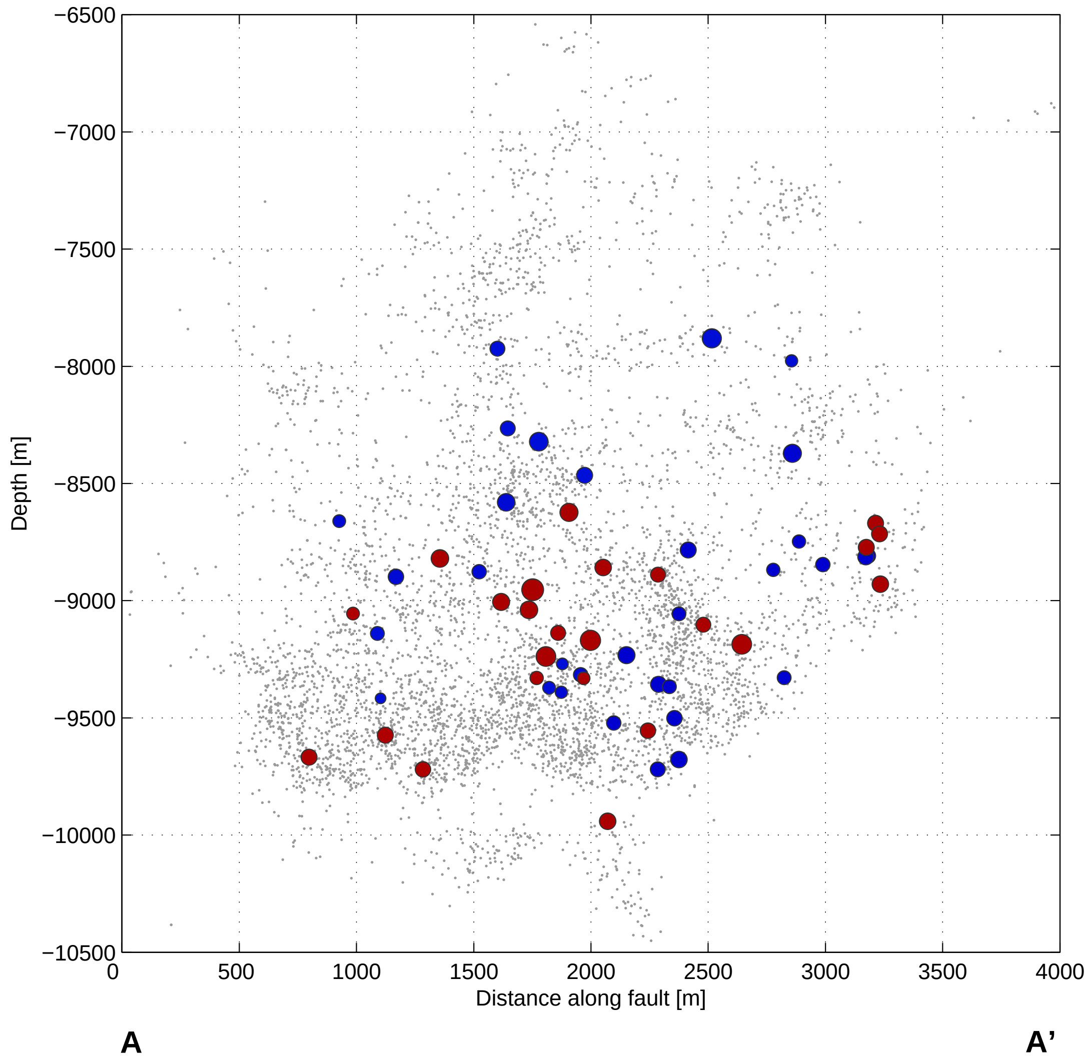

Figure 2: Distribution of hypocenters in the 2000 (blue) and 2008 (red) swarms along the fault plane (viewing from ENE to WSW). Size of the circles corresponds to the seismic moment. Hypocenters located by hypoDD location program (Waldhauser and Ellsworth, 2000) in 1D inhomogeneous velocity model (Málek et al., 2000). Locations provided by Hana Čermáková.

5 Results

5.1 Absolute methods

As a solution of the inversion of P-wave spectra I obtained for each event the corner frequency f_c and N values of Q (having the set of N stations for each event). The seismic moment M_0 was determined from the low-frequency spectral amplitudes and was not inverted for. Applying (4) to the corner frequency f_c I got an approximate source radius. Overview of scaling of the rupture radius r with seismic moment M_0 is in Fig. 3. The resulting dependence of r on M_0 is approximated by

$$r = 0.167\, M_0^{0.202} \tag{17}$$

with the correlation coefficient 0.78. The rupture radii range from 2 to 170 m and the static stress drops between about 1 MPa and 130 MPa. The error bars in Fig. 3 are for the rupture radii and are calculated from the uncertainty $err\ f_c$ which is determined by the corner frequencies at the 5% increase of the *SR* with respect to its minimum. These error estimates are most probably underestimated but at least provide the same relative measure of uncertainty for all events. A rather weak scaling of the source radius with seismic moment in the form $M_0^{0.20}$ is obtained, which points to the deviation from the constant stress-drop model that would correspond to the scaling in the form $M_0^{1/3}$.

The joint inversion applied to P waves allowed to stabilize the estimates of quality factor *Q* and also to test its uncertainty by using the Jackknife test while omitting individual stations.

Application of the absolute method to S waves was made with aim to find the relations for f_c, M_0 and *Q* between the P and S waves. The results are summarized in conclusions.

The Snoke's approach gives an interesting result. The linear regression between the $f_c(J)$ and the $f_c(inv)$ obtained from the inversion of P waves has form

$$f_c(J) = 0.967\, f_c(inv) + 2.317 \tag{18}$$

with correlation coefficient 0.986 if the attenuation correction is applied. Therefore if we know the correct Q factors the corner frequency could be calculated directly from the corrected data without the inversion process.

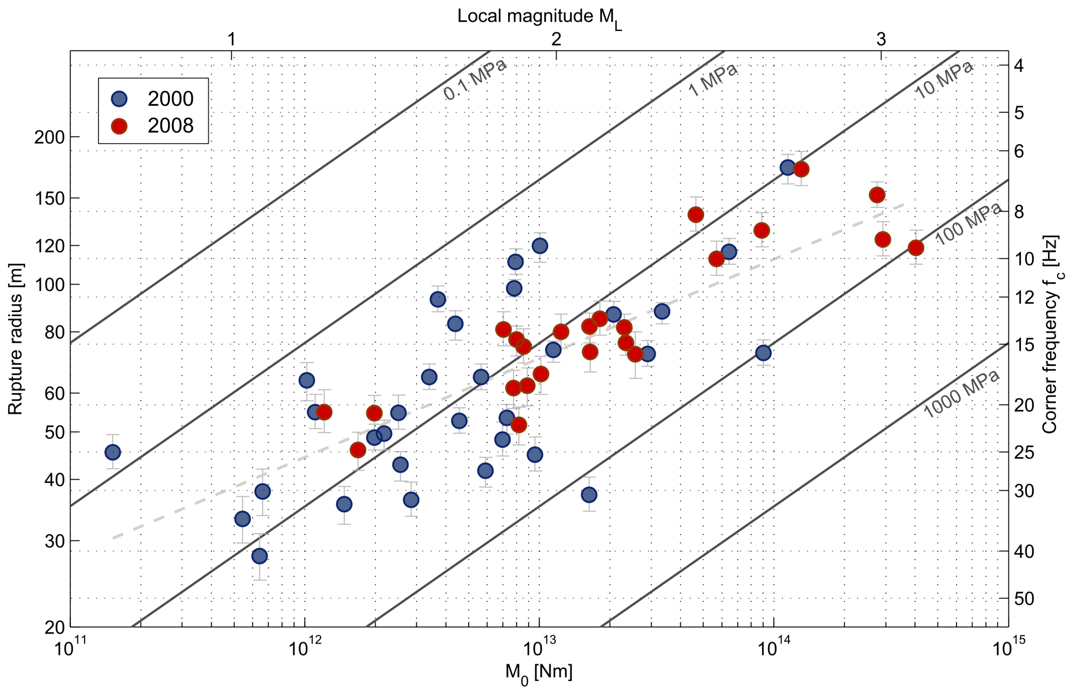

Figure 3: Dependence of the corner frequencies (right axis) and the rupture radii (left axis) on seismic moment obtained by spectra inversion method. The error bars are for rupture radii and are derived from the 5 % uncertainty of the f_c (flatness of the residual function). Blue circles are events from the 2000 swarm and red circles events from the 2008 swarm. The gray dashed line is regression between r and M_0 for all the events together. The upper magnitude axis is scaled according to relation found from regression of $\mathrm{M_L}$ and M_0.

5.2 Relative methods

The relative methods were applied (to P waves only) with aim to eliminate the unknown properties of material between the source and station, i.e. the attenuation effects along the ray path. Even though the relative methods should give more realistic estimations of corner frequencies than the absolute methods, the uncertainties of the relative methods are higher probably due to numerical instabilities in the spectra division and more flat residual function consequently.

5.3 Scaling relations

The relation of seismic moment to local magnitude (Fig. 4)

$$\log M_0 = 1.37\,\mathrm{M_L} + 10.4 \tag{19}$$

was obtained from the regression of the seismic moment M_0 in log scale and the local magnitude $\mathrm{M_L}$ (the correlation coefficient was 0.92). The obtained scaling factor of 1.37 is significantly smaller than 1.5 present in definition of the moment magnitude M_w that was derived for a constant stress drop (Kanamori, 1977), which gives an independent indication of the non-self-similarity of the analyzed swarm earthquakes. In Fig. 4 an empirical relation $\log M_0 = 1.05\,\mathrm{M_L} - 11.3$ obtained by Hainzl and Fischer (2002) from analysis of catalogue of the 2000 swarm in WB is also shown. The scaling factor 1.05 is even lower than received in this study but is similar to relation of Grosser et al. (1986).

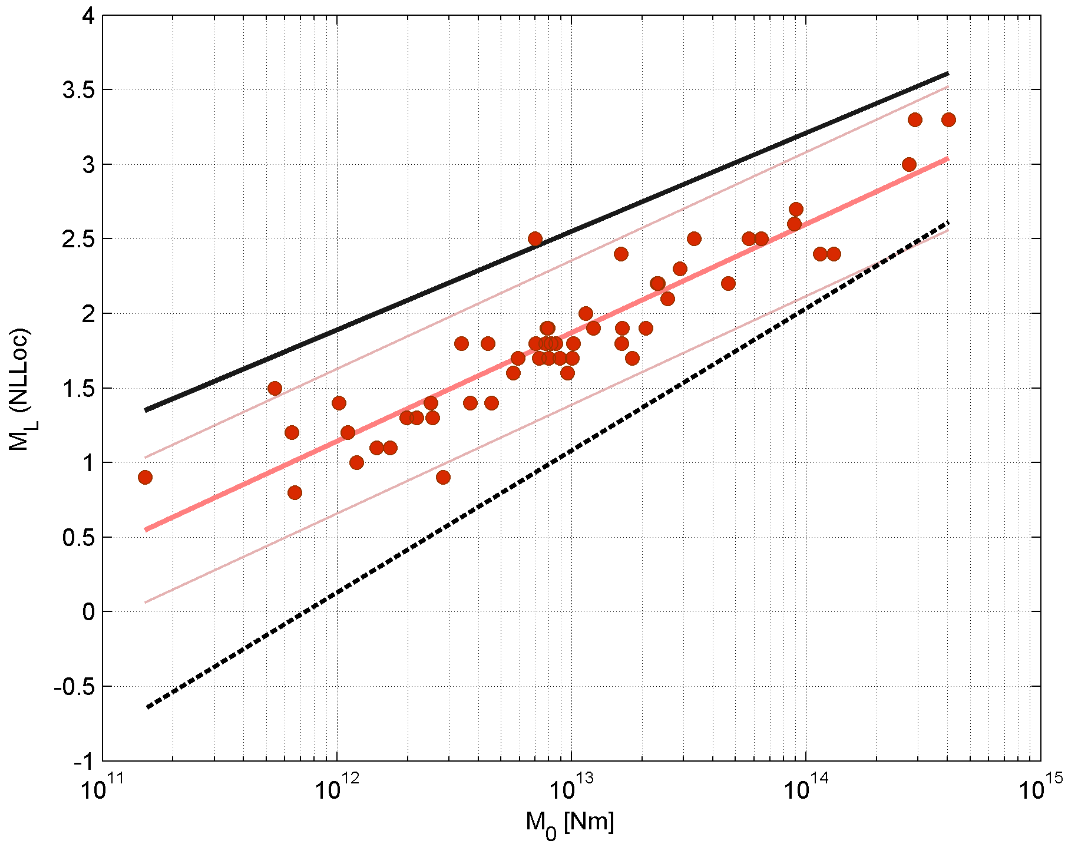

Figure 4: Linear regression between the local magnitude M_L and the seismic moment M_0 in log scale. The 95 % confidence interval is bounded by solid red lines. The relation for moment magnitude $\mathrm{M}_w = {}^2\!/\!_3 \log M_0 - 6.03$ (Hanks and Kanamori, 1979) is plotted by solid black line. Similar empirical relation for the local magnitude obtained by (Hainzl and Fischer, 2002) is plotted by the dashed black line.

5.4 Comparison with results in other studies

Comparison of relations between the corner frequency f_c and the seismic moment M_0, obtained by the absolute approach from P waves is given in Fig. 5. Here I would like to mention that M_0 and f_c are almost independent on the selected model of the source (only f_c is conditioned by the selected high-frequency fall off). Using the axis scaling as used in Fig. 5 the stress drop from this study seems to be almost constant over the whole dataset, just having higher values. It is worth noting that even though some of the results show decrease of the stress drop with the decreasing seismic moment (e.g. Urbancic and Young, 1993; Ide et al., 2003) the overall trend across all studies points to self-similarity of the earthquakes. It is still question whether the leveling-off of f_c with decreasing M_0 has a physical reason or it is just an artifact of the limited data or approach (limited bandwidth, attenuation, simplified model of the source). Keeping in mind the uncertainties of f_c and of $\Delta\sigma$ consequently only the trends are worthy of comparison.

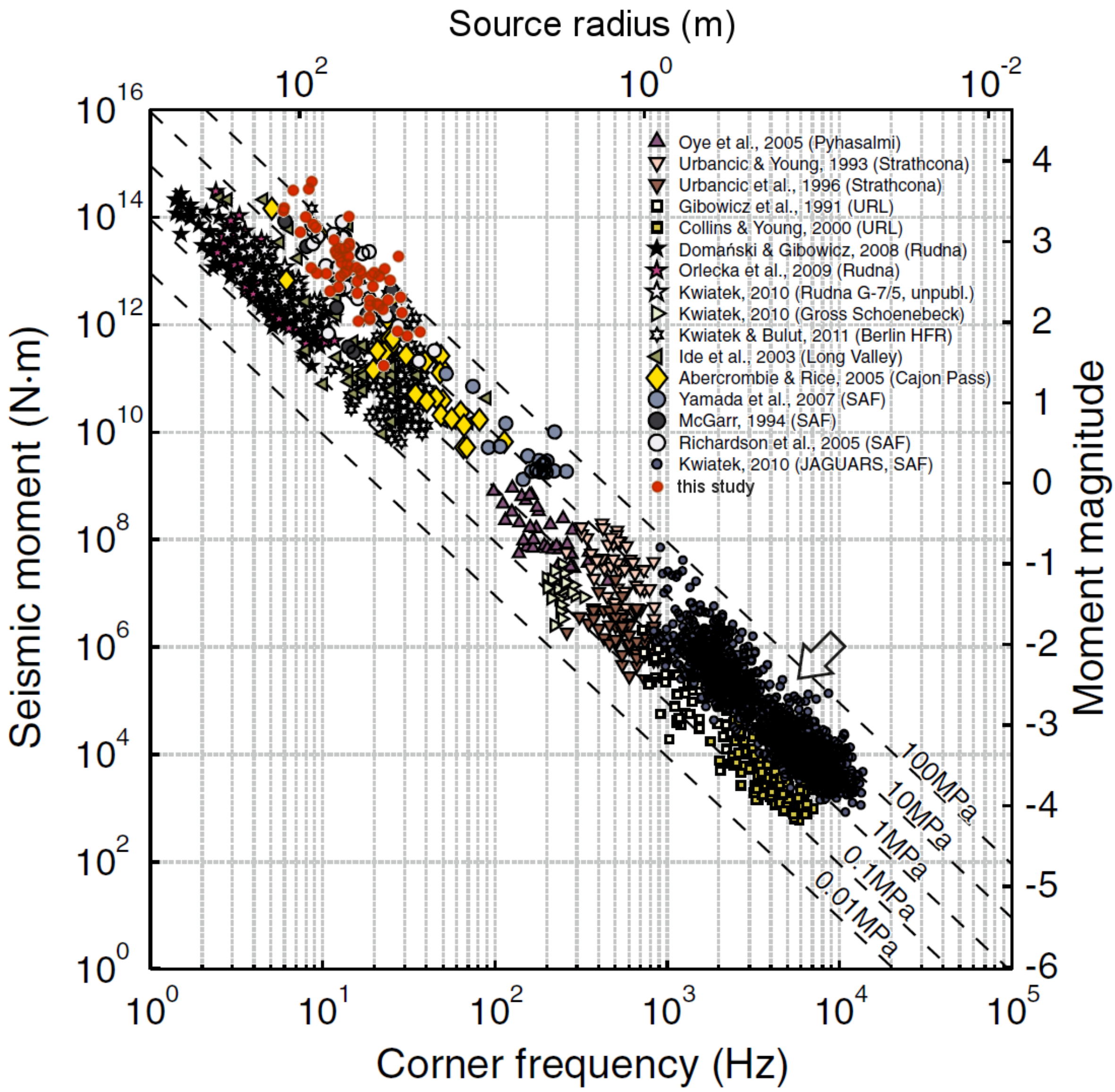

Figure 5: Comparison of f_c and M_0 obtained from analysis of P waves (red circles) with other results (adopted from Kwiatek et al., 2011).

The second, and probably the main, criterion for the earthquake self-similarity assessment is the effectivity of the radiated energy – the apparent stress $\Delta\sigma_a$. Figure 6 shows comparison of the apparent stress with results in other studies. As the apparent stress increases with the seismic moment I could deduce that the small earthquakes are less efficient in radiating the energy than the bigger ones, what supports the non-self-similarity (e.g. Aki, 1967; Kanamori and Anderson, 1975; Mayeda et al., 2005). The obvious decrease of $\Delta\sigma_a$ with decreasing M_0 is observed almost in all studies. It is interesting that this trend occurs almost in each separate study but as a whole $\Delta\sigma_a$ seems to be within a band of "constant" values. Again, it is a question whether the weaker events really radiate less energy or if it is a consequence of a hidden artifact.

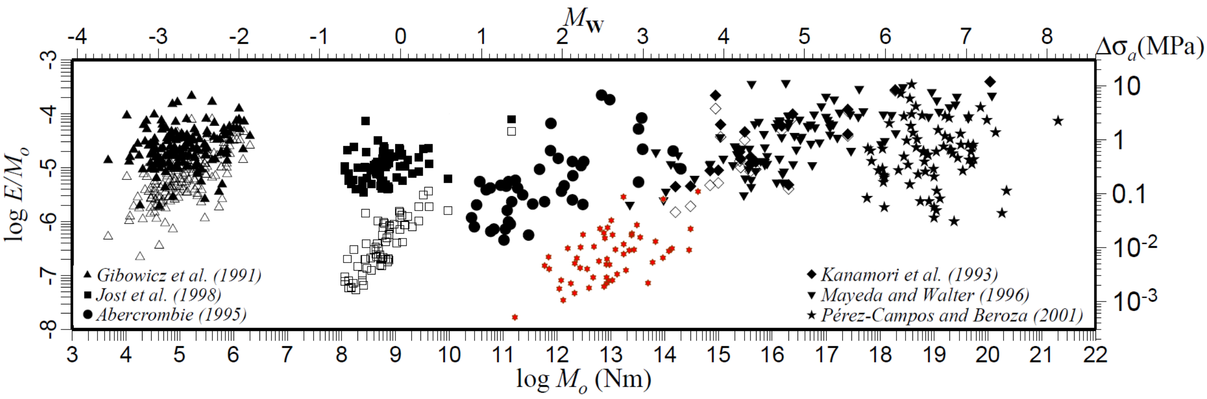

Figure 6: Comparison of scaled energy with other studies (adopted from Ide and Beroza, 2001).

5.5 Effects influencing the source parameter estimation

In the thesis I analyzed other effects which could influence the obtained source parameters, namely the effect of quality factor Q, the radiation pattern corrections $\Re$, the free surface correction F_{surf} and also possible effect of directivity of the source.

The most important role in estimation of the corner frequency plays the quality factor Q because of the influence of the slope of the high-frequency fall off. Therefore the absolute joint inversion over multiple events should give better conditioned results. The EGF method should also remove the path effects but it is not applicable to all events.

The radiation pattern correction $\Re$ influence the estimate of the seismic moment M_0 mainly. In the thesis I compared M_0 obtained with an average radiation pattern correction $\langle\Re\rangle$ to the results while using azimuth-dependent $\Re$. I found that for the P waves $\langle\Re\rangle$ could give M_0 with lower uncertainty than when using $\Re$ from all stations because of the low values of $\Re$ close to the nodal lines.

The free surface correction F_{surf} in case of sub-vertical incidence angle does not influence the amplitudes (or M_0) because of the almost constant value 2.0.

I made an attempt to discriminate the effects of directivity in the source but I found that if there are any such effects they could be easily hidden/masked by the effects of attenuation.

6 SEISMON

SEISMON is a GUI MATLAB software initially developed by Stefan Mertl, TU in Vienna (Mertl and Hausmann, 2009). I joined the SEISMON project development in 2010 and since then the code is developed and modified by team of people at Seismological Department of Institute of Geophysics AS CR for processing of data mainly from the WEBNET seismic network.

The analysis of the source parameters was done mainly in SEISMON and a new tools were developed for this purpose (Fig. 7). It allows to fit the basic spectral models of the source to the observed data and invert for the selected quantities (Fig. 8).

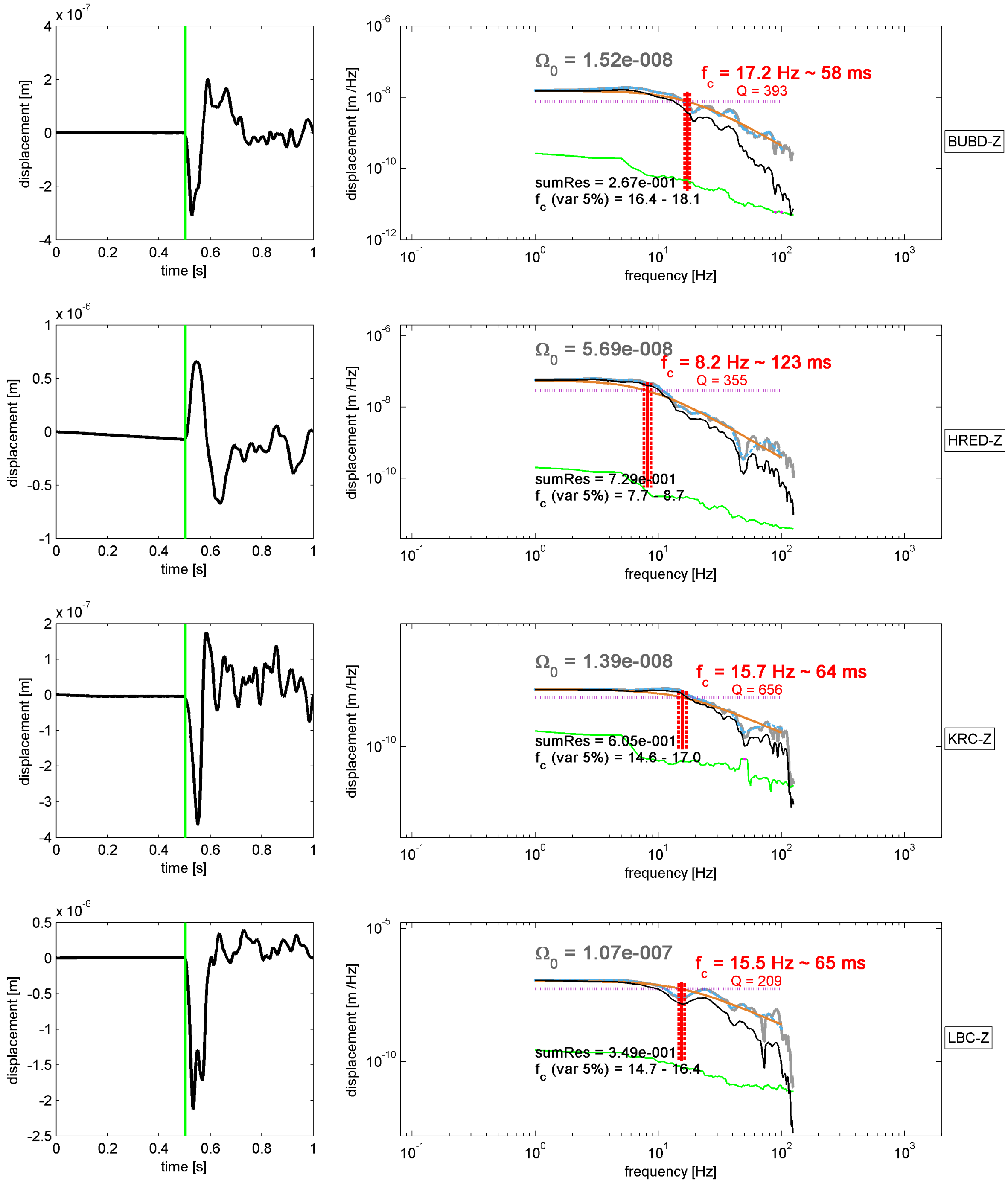

Figure 7: Spectral analysis in SEISMON. Example of absolute source parameter inversion from P-wave displacement (left) at four stations. Results of inversion at individual stations are written in spectra (right) in red color.

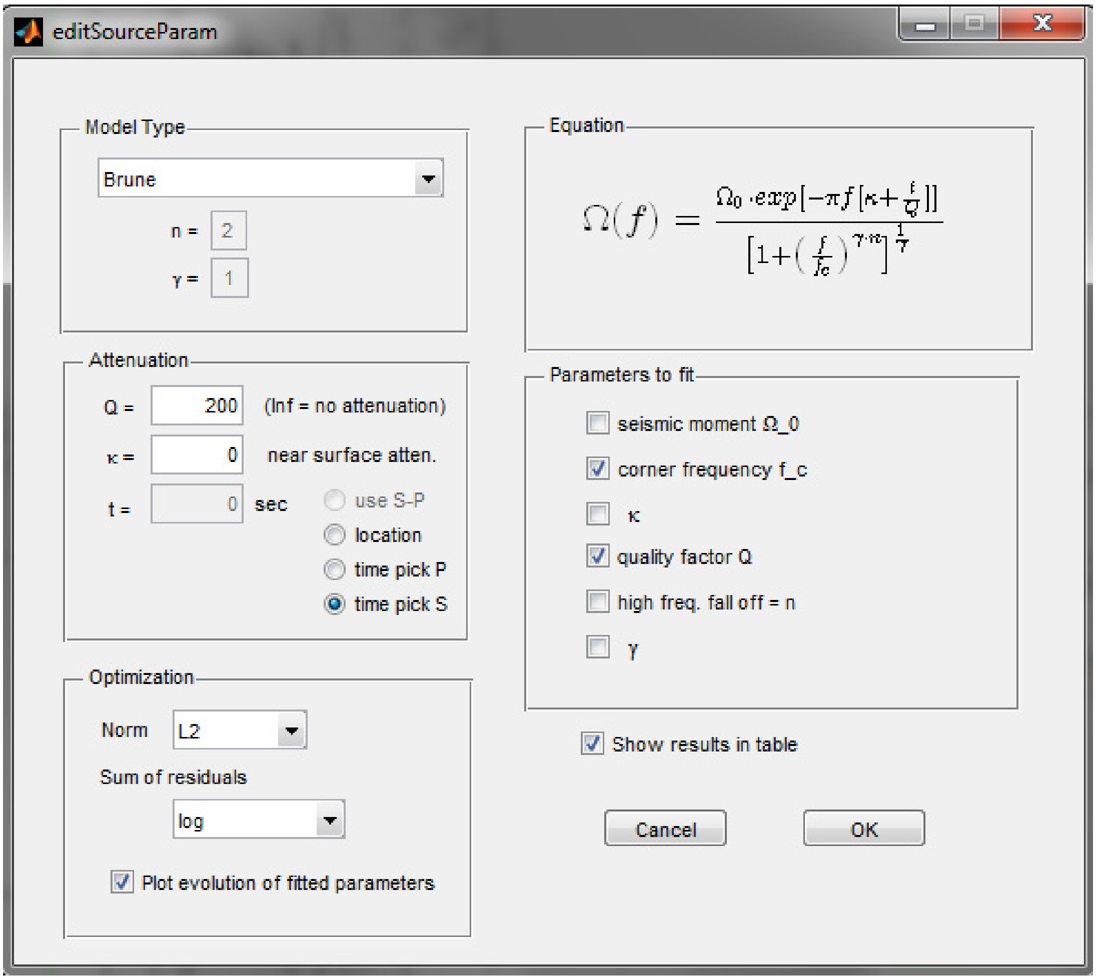

Figure 8: GUI window in SEISMON tool for setting up the inversion parameters.

7 Conclusions

I studied source parameters of the West Bohemia/Vogtland earthquake swarms 2000 and 2008 in the frequency domain and analyzed 56 events in the magnitude range $\mathrm{M_L}$ from 0.8 to 3.3 that were evenly distributed along the fault plane. Direct P waves and S waves from 3 to 21 stations at epicentral distances from 0 to 30 km were used. I applied the absolute and relative approach to invert for source parameters. The absolute approach allowed to retrieve the seismic moment M_0 and corner frequency f_c and was applied to P waves (inversion of individual events and the joint inversion of all events together) and to S waves (inversion of individual events only). The relative approach was applied with aim to obtain more precise corner frequency by separation of the source from the material-dependent attenuation. The inversion of S waves, the joint inversion and the relative approach was applied to a subset of 24 events only from the 2008 earthquake swarm because of much better focal sphere coverage by stations than in 2000. From the absolute approach I obtained the quality factor Q as a byproduct. In all approaches the same Brune's circular model of the source with ω^{-2} high-frequency falloff was used with assumptions of a quasidynamic behavior after Madariaga (1976). The main conclusions can be summarized as follows:

- The corner frequencies of the $\mathrm{M_L}$ 0.8–3.3 events range from 6 to 40 Hz, which corresponds to the rupture radii in the range 28 to 150 m for seismic moments from 1.5e11 to 4e14 Nm.
- To verify the precision of M_0 obtained from the absolute approach and to test the influence of the radiation pattern correction I evaluated M_0 by the AMT method. The radiation pattern correction influences mainly estimates at stations close to the nodal lines of the focal mechanisms. However, even if only the mean radiation pattern correction is applied the absolute and AMT methods give similar results; differences of 0.3 in the log scale which correspond to usual uncertainties of $\mathrm{M_L}$.
- An alternative method of Snoke was applied to P waves to determine the radiated energy $E(J)$ and the corner frequency $f_c(J)$ from the integral of the velocity squared spectra *J*. This method gave very similar f_c as obtained from the absolute inversion if the correction for attenuation is applied. This good fit qualifies the more simple Snoke's method to be applicable for routine determination of the static source parameters of small earthquakes in case that the quality factor Q is known and applied.
- The absolute joint inversion for f_c and Q over 24 events and all available stations simultaneously for P waves resulted in Q-factor between 117 and 400 at different stations.The Q from the joint inversion is more stable than from inversion of individual events.

- Comparison of methods for inversion of f_c is in favor of the absolute methods compared to the relative ones. Even though the relative methods should separate the source term effectively the uncertainty of this methods is higher. Especially the joint absolute inversion allows to estimate the uncertainties of f_c more reliably.

- Comparison of results of the absolute methods applied to P and S waves gives: f_c^α/f_c^β is most often between 1.0 and 1.5; median of Q_S/Q_P ratios is 1.32 instead of expected 0.4; the ratio $M_0^P/M_0^S = 1.51$ would need $\alpha/\beta = 1.9$ which is unrealistically high. Explanation is not provided but results are similar to other studies.

- Scaling of f_c with M_0 shows an exponent of -0.202, which in the absolute sense is smaller than -0.33 that is expected for the constant stress drop model. The stress drops $\Delta\sigma$ range between 0.7 to 138 MPa with tendency to a higher stress drops for the larger events. But taking into account the high uncertainty of $\Delta\sigma$ (which is still very likely underestimated) these values should not be used for any further interpretations.

- Apparent stress $\Delta\sigma_a$ ranges over three orders from 0.3 kPa to 0.4 MPa, which supports finding that the West–Bohemia swarm earthquakes are not self-similar.

The simple model of the source can never adopt all the aspects of the real source and the values represent a very rough estimates in terms of orders only. Comparison between similar individual studies even using the same source model can differ in the absolute sense because of the different numerical implementation and the specific signal operations. Application of either absolute or relative spectral methods for estimation of the source parameters of microearthquakes or weak earthquakes generally must be done with knowledge of the possible uncertainties. Interpretation of the results must be always done with respect to the used model of the source.

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