Ocean induction modelling: Method, benchmarks and predictions of DEBOT and LSG signatures

Swarm + Oceans

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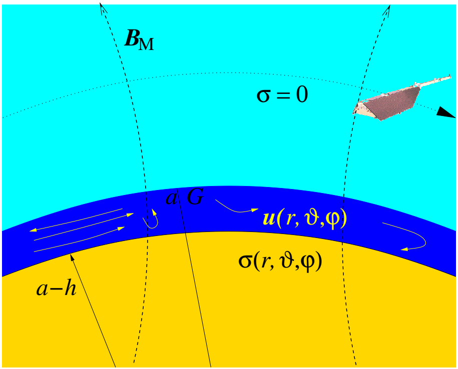
Outline

- Magnetic field induced by 3D ocean flow
- Benchmarks
- Magnetic field generated by the DEBOT model
- Magnetic field generated by the LSG model
- WP3000 Status

Magnetic field induced by 3D ocean flow

Problem description



Magnetic field induced by 3D ocean flow

Classical formulation

$$\boldsymbol{B} = \boldsymbol{B}(\boldsymbol{r};t) \quad \boldsymbol{u} = \boldsymbol{u}(\boldsymbol{r};t) \quad \rho = \rho(\boldsymbol{r})$$

$$\frac{1}{\mu_0}\nabla \times (\rho\nabla \times \boldsymbol{B}) - \nabla \times (\boldsymbol{u} \times \boldsymbol{B}) + \frac{\partial \boldsymbol{B}}{\partial t} = 0$$

$$\boldsymbol{B}(\boldsymbol{r};t) = \boldsymbol{B}_M(\boldsymbol{r};t) + \boldsymbol{b}(\boldsymbol{r};t)$$

$$|\boldsymbol{B}_M| \gg |\boldsymbol{b}| \quad \left|\frac{\partial \boldsymbol{B}_M}{\partial t}\right| \ll \left|\frac{\partial \boldsymbol{b}}{\partial t}\right| \quad \boldsymbol{B}_M(\boldsymbol{r};t) = -\nabla U_M(\boldsymbol{r};t)$$

$$\frac{1}{\mu_0}\nabla \times (\rho\nabla \times \boldsymbol{b}) + \frac{\partial \boldsymbol{b}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B}_M)$$

$$\boldsymbol{b}(\boldsymbol{r};t) = -\nabla u(\boldsymbol{r};t)\big|_{r=a} \quad \nabla^2 u = 0\big|_{r\geq a} \quad \lim_{r\to\infty} u(\boldsymbol{r};t) = 0$$

Magnetic field induced by 3D ocean flow

Crank-Nicolson scheme

Courant-Friedrichs-Lewy criterion restricts the explicit schemes
$\Delta t < \mu_0\, \sigma\, \Delta x^2 \approx 10^{-6}\,\mathrm{H/m}\; 10^{-3}\,\mathrm{S/m}\; 10^{6}\,\mathrm{m}^2 \approx 10^{-3}\,\mathrm{s}$

$$\begin{aligned} \mathbf{A}\cdot\mathbf{x}_{i+\frac{1}{2}} &= \mathbf{b}_{i+\frac{1}{2}}, \\ \mathbf{x}_{i+1} &= 2\,\mathbf{x}_{i+\frac{1}{2}} - \mathbf{x}_i. \end{aligned} \qquad \begin{aligned} \mathbf{A} &= \left(\frac{2}{\Delta t}\mathbf{M} + \mathbf{R} + \mathbf{B}\right), \\ \mathbf{b} &= \mathbf{l}_{i+\frac{1}{2}} + \frac{2}{\Delta t}\mathbf{M}\cdot\mathbf{x}_i, \end{aligned}$$

except for ($k = K+1$), where boundary conditions are used

$$\begin{aligned} B_{jm}^{0\,K+1} &= 0, \\ -\frac{1}{2j+1}\left[B_{jm}^{-1\,K+1} + (j+1)\,B_{jm}^{1\,K+1}\right] &= G_{jm}^{(e)} = 0, \\ \Lambda_{jm}^{K+1} &= 0. \end{aligned}$$

Magnetic field induced by 3D ocean flow

Block-tridiagonal matrix

$$\begin{pmatrix} \mathbf{D}_1 & \mathbf{U}_1 & & & & & \\ \mathbf{L}_2 & \mathbf{D}_2 & \mathbf{U}_2 & & & & \\ & \ddots & \ddots & \ddots & & & \\ & & \mathbf{L}_k & \mathbf{D}_k & \mathbf{U}_k & & \\ & & & \ddots & \ddots & \ddots & \\ & & & & \mathbf{L}_K & \mathbf{D}_K & \mathbf{U}_K \\ & & & & & \mathbf{L}_{K+1} & \mathbf{D}_{K+1} \end{pmatrix} \cdot \begin{pmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \vdots \\ \boldsymbol{x}_k \\ \vdots \\ \boldsymbol{x}_K \\ \boldsymbol{x}_{K+1} \end{pmatrix} = \begin{pmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \\ \vdots \\ \boldsymbol{b}_k \\ \vdots \\ \boldsymbol{b}_K \\ \boldsymbol{b}_{K+1} \end{pmatrix}$$

$$\dim \boldsymbol{x}_k = \dim \boldsymbol{b}_k = 4\,J\,(J+2)$$

$$\dim \mathbf{L}_k = \dim \mathbf{D}_k = \dim \mathbf{U}_k = [4\,J\,(J+2)]^2$$

$\mathbf{D}_k$ real, symmetric, indefinite; $\mathbf{D}_{K+1}$ is non-symmetric

for 1-D layers they further split to block-diagonal matrices of 1×1 and 3×3 blocks

Magnetic field induced by 3D ocean flow

Generalized Thomas algorithm

- Forward phase

$$\tilde{\mathbf{D}}_1 = \mathbf{D}_1$$
$$\tilde{\boldsymbol{b}}_1 = \boldsymbol{b}_1$$

For $k = 2, \ldots, K+1$

$$\tilde{\mathbf{D}}_k = \mathbf{D}_k - \mathbf{L}_k \cdot \tilde{\mathbf{D}}_{k-1}^{-1} \cdot \mathbf{U}_{k-1}$$
$$\tilde{\boldsymbol{b}}_k = \boldsymbol{b}_k - \mathbf{L}_k \cdot \tilde{\mathbf{D}}_{k-1}^{-1} \cdot \tilde{\boldsymbol{b}}_{k-1}$$

- Backward phase

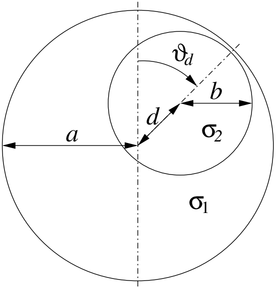
$$\boldsymbol{x}_{K+1} = \tilde{\mathbf{D}}_{K+1}^{-1} \cdot \tilde{\boldsymbol{b}}_{K+1}$$

For $k = K, \ldots, 1$

$$\boldsymbol{x}_k = \tilde{\mathbf{D}}_k^{-1} \cdot \left(\tilde{\boldsymbol{b}}_k - \mathbf{U}_k \cdot \boldsymbol{x}_{k+1}\right)$$

Benchmarks

Nested spheres

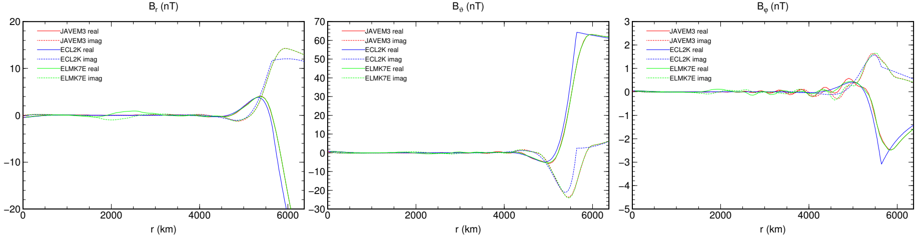


$$a = 6371\,\mathrm{km}$$
$$\sigma_1 = 0.01\,\mathrm{S/m}$$
$$b = 3500\,\mathrm{km}$$
$$\sigma_2 = 1\,\mathrm{S/m}$$
$$d = 2200\,\mathrm{km}$$
$$\vartheta_d = 40^\circ$$
$$\varphi_d = 35^\circ$$

- excited by dipolar external field at period $T = 4\,\mathrm{days}$
- tests the matrix $\mathbf{A}$ for a fully 3-D conductivity model

Benchmarks

Nested spheres



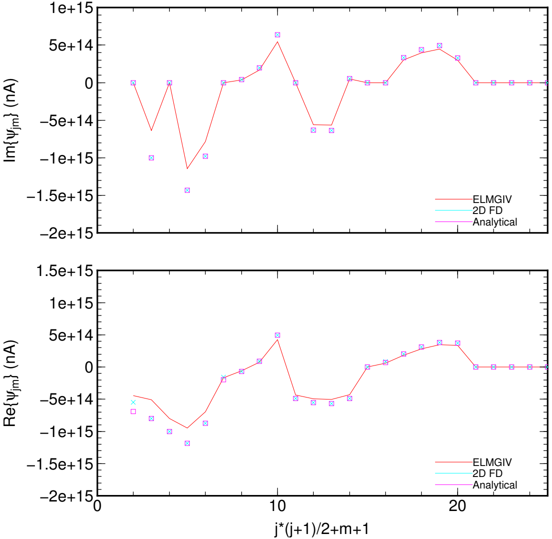
Benchmarks

Shallow-water approximation

- 2-D FD discretization of shallow water approximation of the induction equation
- stationary solution ($\frac{\partial \boldsymbol{b}}{\partial t} = 0$)
- simplified spatial operators
- global ocean, $h = 3\,\mathrm{km}$, $\sigma = 3.5\,{}^{\mathrm{S}}/_{\mathrm{m}}$
- insulating mantle
- velocities of ocean flows described by spherical harmonics of degree
  - a) 1–4
  - b) 10
  - c) 20
- dipolar main field

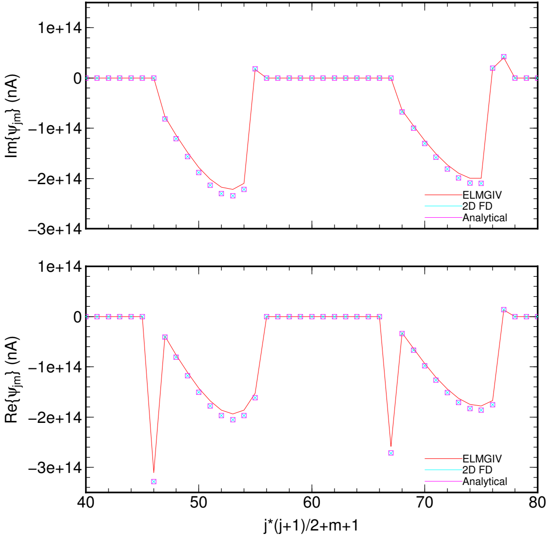
Benchmarks

Shallow-water approximation



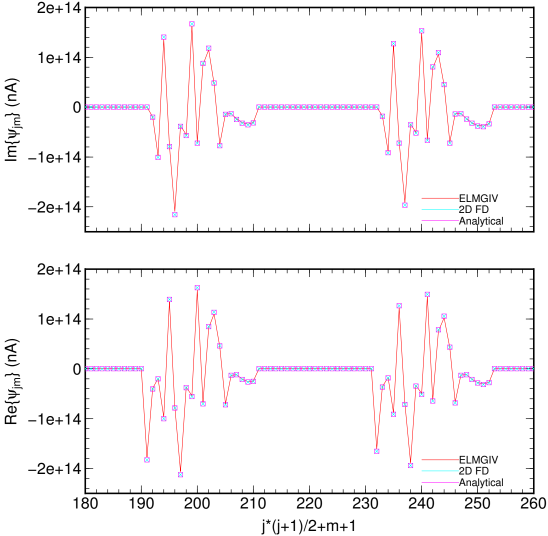
Benchmarks

Shallow-water approximation



Benchmarks

Shallow-water approximation



Benchmarks

1-D time-domain benchmark

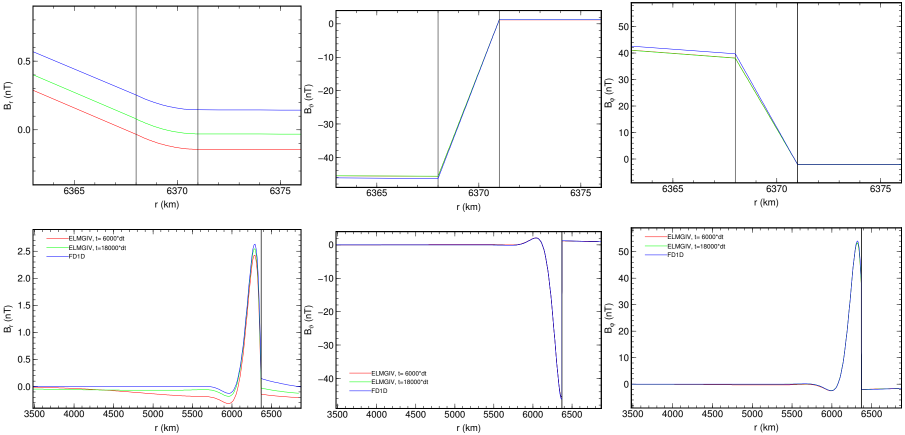
- Toroidal harmonic Lorentz force up to degree and order 4

$$L_{jm}^{(0)} = A_{jm}\cos\omega t + B_{jm}\sin\omega t + \mathrm{i}\left(C_{jm}\cos\omega t + D_{jm}\sin\omega t\right)$$

- Homogeneous mantle, $\sigma = 1\,\mathrm{S}/\mathrm{m}$
- Homogeneous global ocean, $\sigma = 3.5\,\mathrm{S}/\mathrm{m}$, $h = 3\,\mathrm{km}$

Benchmarks

1-D time-domain benchmark



Benchmarks

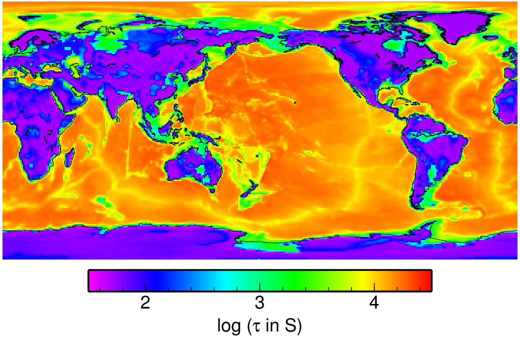
Effects of resolution

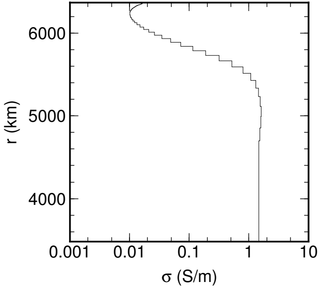
- DEBOT flow model, $30' \times 30'$
- $A_H = 1.5\,10^5\ \mathrm{m^2/s}$, $r = 3\,10^{-3}$
- IGRF11 main field
- 1-D mantle conductivity (Kuvshinov & Olsen 2006) and surface conductance map (Everett et al. 2003)

Run	J	Δt	K	k_{ocean}
A	40	0.1 hrs	71	5
B	40	0.2 hrs	71	5
C	20	0.5 hrs	71	5
D	40	0.5 hrs	65	2
E	40	0.5 hrs	71	5
F	40	0.5 hrs	81	10
G	60	0.5 hrs	71	5

Benchmarks

Effects of resolution

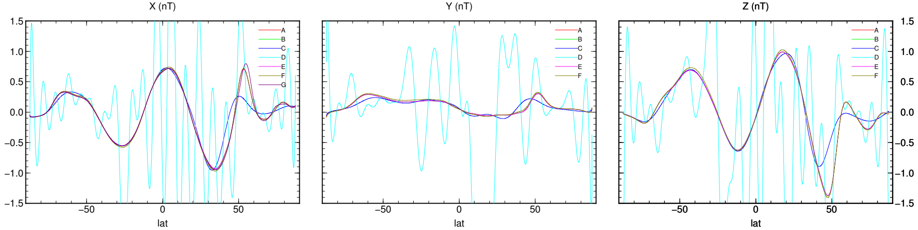




conductivity model

Benchmarks

Effects of resolution



Magnetic signatures of DEBOT model along the Swarm A track No. 1755.5 for various resolution settings.

Benchmarks

Velocity interpolation

- Arakawa grids use different grids for individual velocity components
- Calculation of $\boldsymbol{L} = \boldsymbol{u} \times \boldsymbol{b}$ requires all components of $\boldsymbol{u}$ on the same grid
- regridding must be fast (repeated at each step with precomputed weights)

Benchmarks

Velocity interpolation

1. Gaussian filter

$$\Theta_{ij} = \Theta\left(\Omega_i, \tilde{\Omega}_j\right) = \arccos\left[\sin\phi_i \, \sin\tilde{\phi}_j + \cos\phi_i \, \cos\tilde{\phi}_j \, \cos(\lambda_i - \tilde{\lambda}_j)\right]$$

$$w_{ij} = \begin{cases} \frac{1}{S_i}\exp(-\frac{\Theta_{ij}^2}{\Theta_0^2}) & \text{for} \quad \Theta_{ij} \leq 3\,\Theta_0 \\ 0 & \text{for} \quad \Theta_{ij} > 3\,\Theta_0 \end{cases}$$

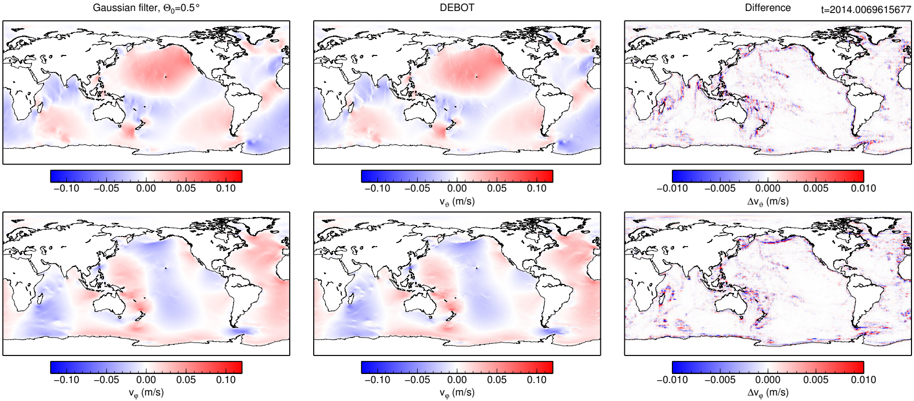
$$\sum_j w_{ij} = 1 \quad \forall i, \qquad u_k(\Omega_i) = \sum_j w_{k,ij} u_k(\tilde{\Omega}_j)$$

2. bilinear (2D) or trilinear (3D) interpolation

$$\begin{aligned} \tilde{r}_j \leq \quad & r_i \quad < \tilde{r}_{j+1} \\ \tilde{\vartheta}_k \leq \quad & \vartheta_i \quad < \tilde{\vartheta}_{k+1} \\ \tilde{\varphi}_l \leq \quad & \varphi_i \quad < \tilde{\varphi}_{l+1} \end{aligned} \qquad u(r_i, \vartheta_i, \varphi_i) = \sum_{\alpha=1}^{8} w_\alpha u_\alpha$$

Benchmarks

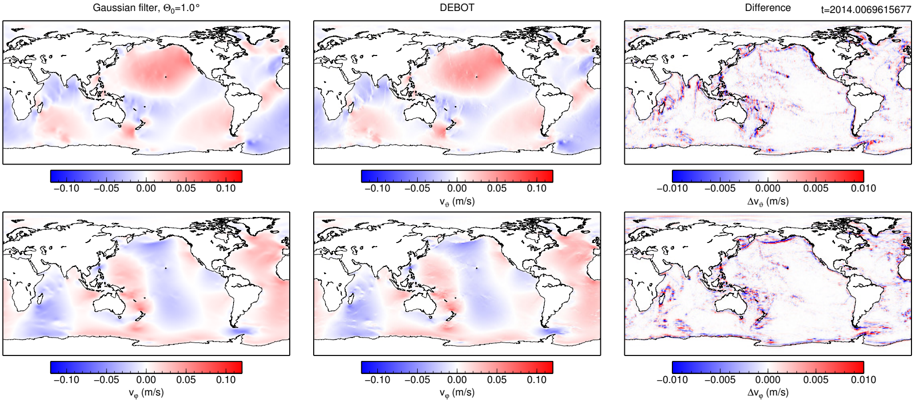
Velocity interpolation



Gaussian velocity interpolation, $\Theta_0 = 0.5^\circ$

Benchmarks

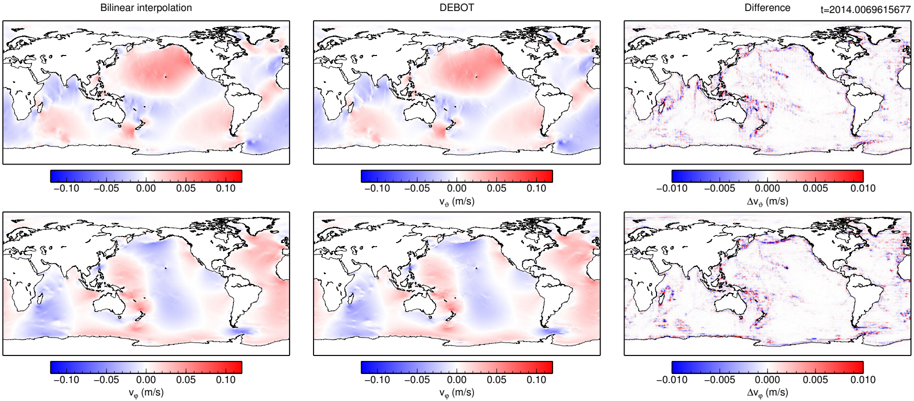
Velocity interpolation



Gaussian velocity interpolation, $\Theta_0 = 1.0°$

Benchmarks

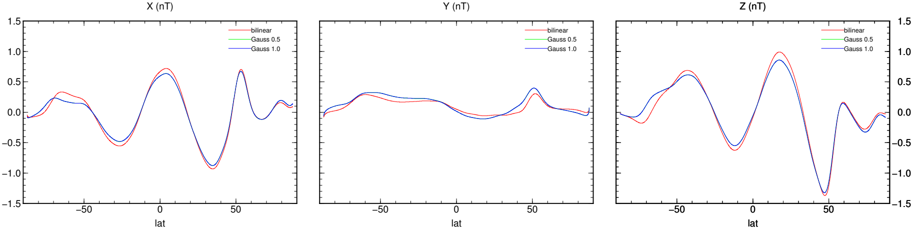
Velocity interpolation



Bilinear velocity interpolation

Benchmarks

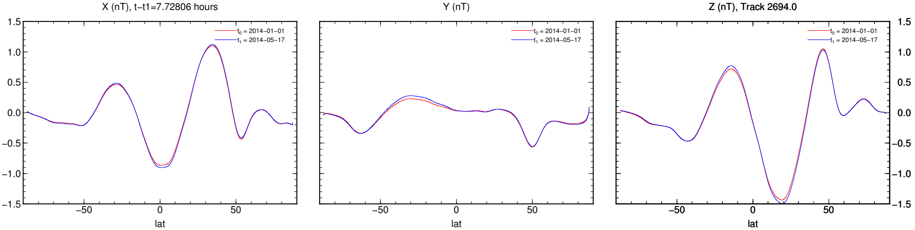
Velocity interpolation



Magnetic signatures of DEBOT model along the Swarm A track No. 1755.5 for various velocity interpolations.

Benchmarks

Transient effect

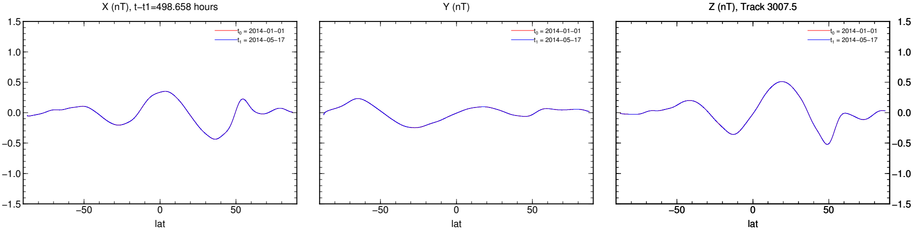


Comparison of two runs with same DEBOT model started at different times

$$
\begin{aligned}
t_0 &= 2014-01-01 \\
t_1 &= 2014-05-17 \\
t - t_1 &= 7.73\,\text{hrs}
\end{aligned}
$$

Benchmarks

Transient effect

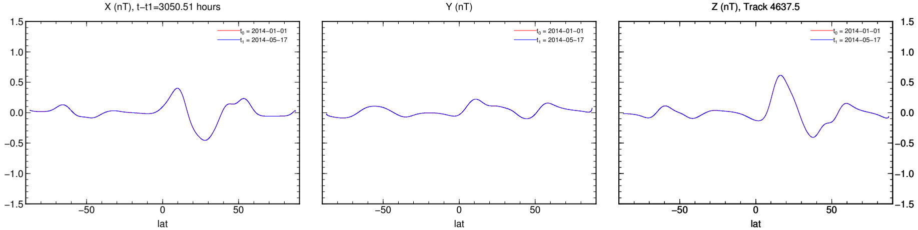


Comparison of two runs with same DEBOT model started at different times

$$\begin{aligned} t_0 &= 2014-01-01 \\ t_1 &= 2014-05-17 \\ t - t_1 &= 498.66\,\text{hrs} \end{aligned}$$

Benchmarks

Transient effect



Comparison of two runs with same DEBOT model started at different times

$$
\begin{aligned}
t_0 &= 2014-01-01 \\
t_1 &= 2014-05-17 \\
t - t_1 &= 3050.51\,\mathrm{hrs}
\end{aligned}
$$

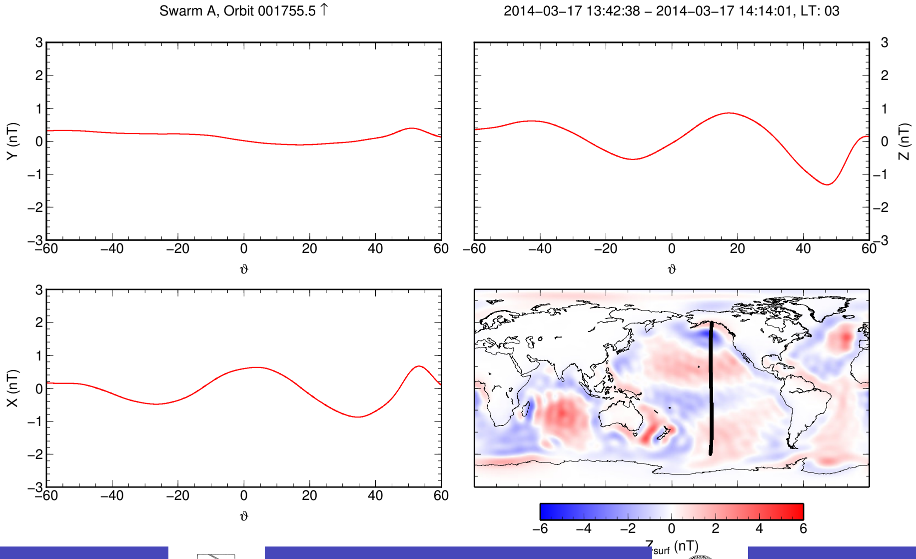
Magnetic field generated by the DEBOT model

Setup

- DEBOT
    - ephemeridal tidal forcing
    - $\Delta t = 30\,\mathrm{min}$
    - $\Delta x = 30' \times 30'$
    - $A_H = 1.5\,10^5\ {}^{\mathrm{m^s}}\!/_{\mathrm{s}}$
    - $r = 3\,10^{-3}$
- EM induction
    - $J = 40$
    - Δt = 30 min
    - 1-D mantle conductivity (Kuvshinov & Olsen 2006) and surface conductance map (Everett et al. 2003)

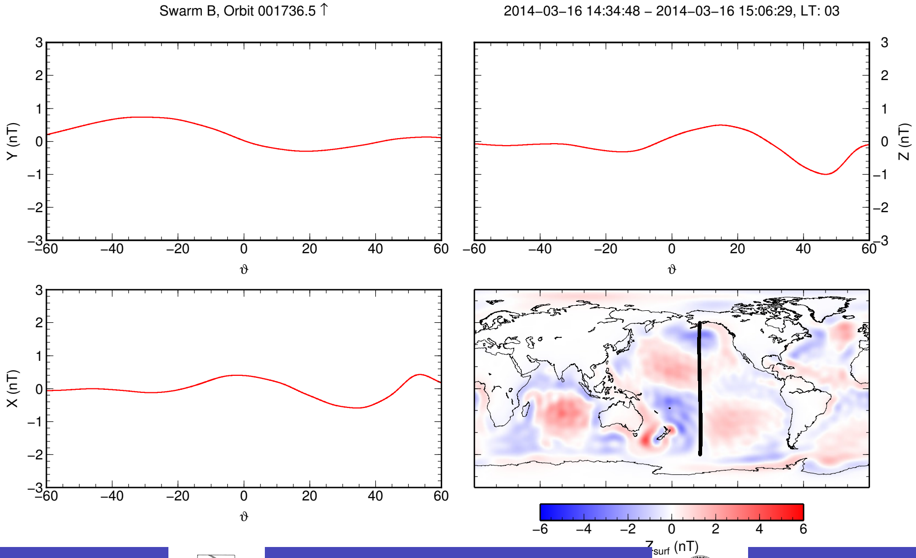
Magnetic field generated by the DEBOT model

Results



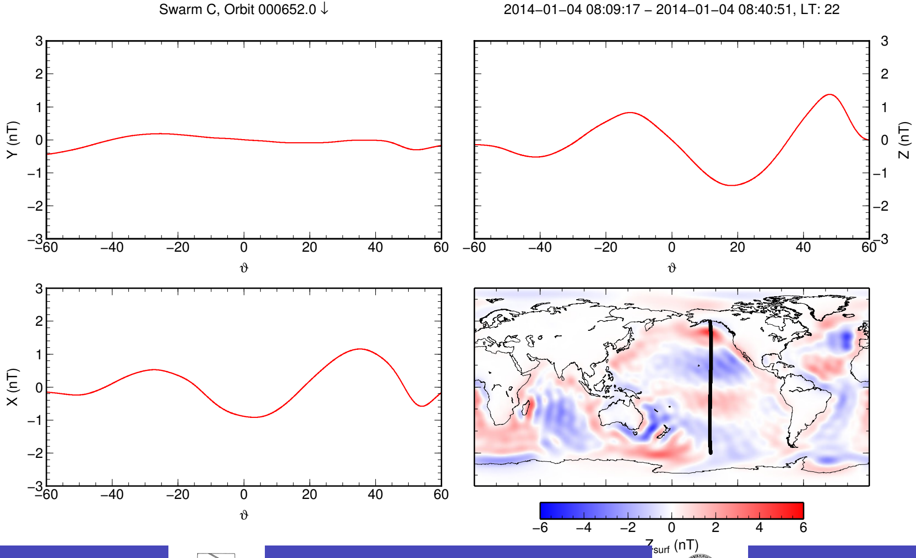
Magnetic field generated by the DEBOT model

Results



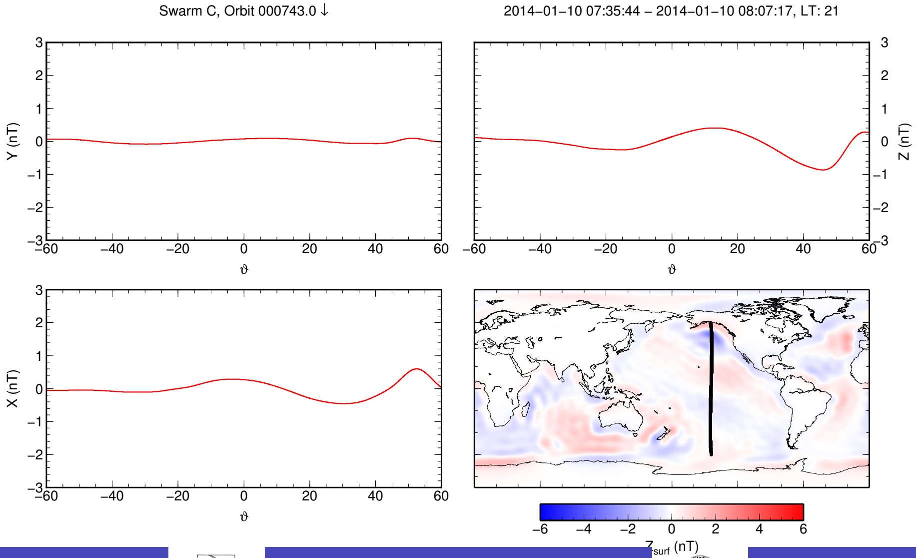
Magnetic field generated by the DEBOT model

Results



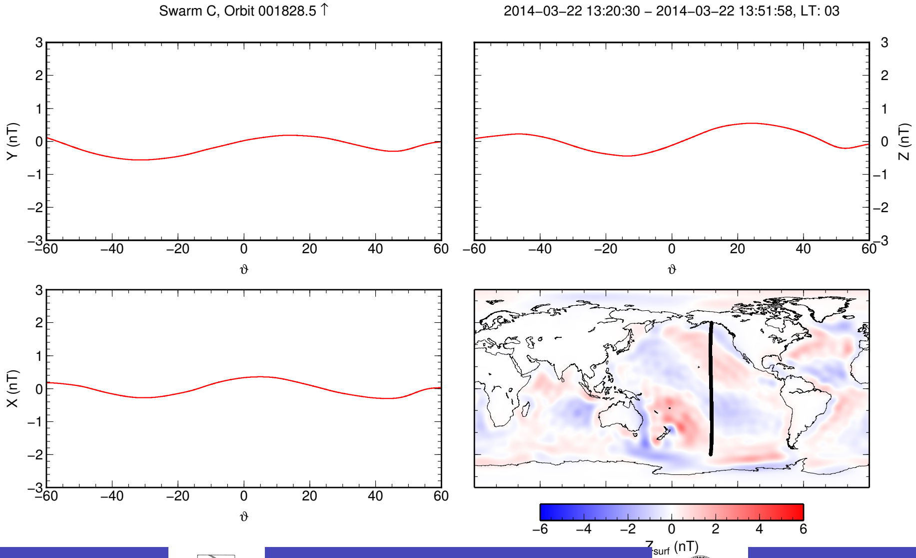
Magnetic field generated by the DEBOT model

Results



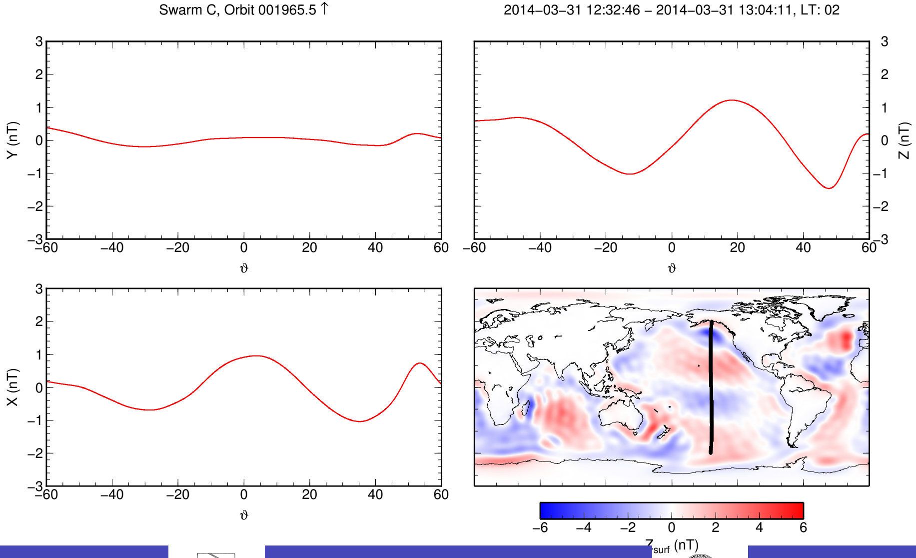
Magnetic field generated by the DEBOT model

Results



Magnetic field generated by the DEBOT model

Results



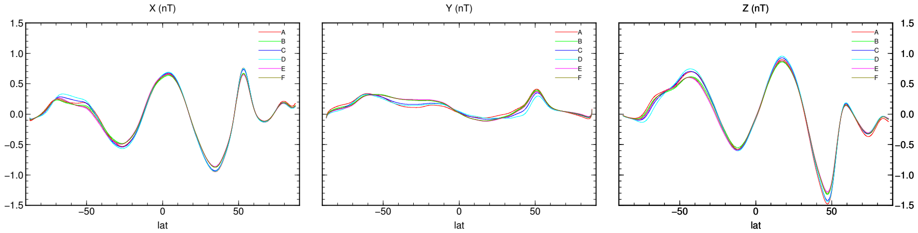
Magnetic field generated by the DEBOT model

Sensitivity to DEBOT parameters

Run	$\Delta\vartheta \times \Delta\varphi$	A_H	ε
A	$20' \times 20'$	$1\,10^4\ \mathrm{m^2/s}$	0.08
B	$30' \times 30'$	$1\,10^4\ \mathrm{m^2/s}$	0.08
C	$30' \times 30'$	$1\,10^4\ \mathrm{m^2/s}$	0.10
D	$30' \times 30'$	$1\,10^4\ \mathrm{m^2/s}$	0.12
E	$30' \times 30'$	$1\,10^5\ \mathrm{m^2/s}$	0.08
F	$30' \times 30'$	$5\,10^4\ \mathrm{m^2/s}$	0.08

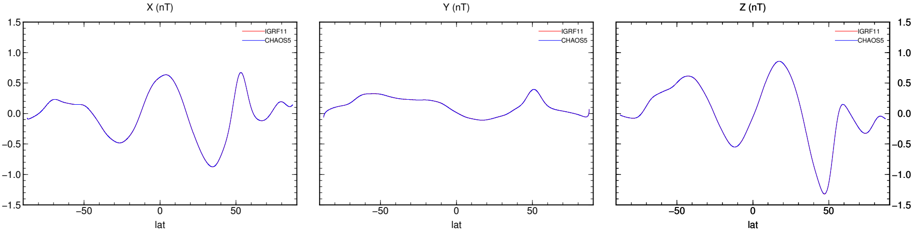
Magnetic field generated by the DEBOT model

Sensitivity to DEBOT parameters



Magnetic field generated by the DEBOT model

Sensitivity to main field model



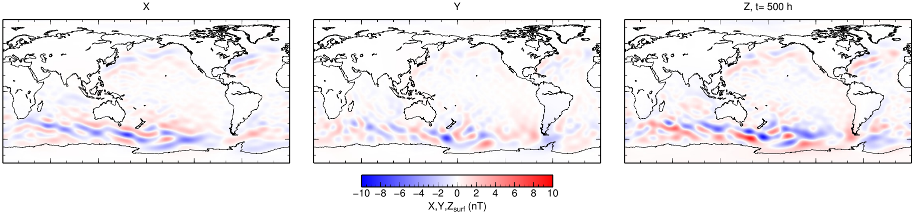
IGRF11 ($J = 13$) vs. CHAOS-5 ($J = 90$)

Magnetic field generated by the LSG model
Setup

- LSG
  - wind forcing, January–April 2013
  - $\Delta x = 60' \times 60'$
- EM induction
  - $J = 40$
  - Δt = 30 min
  - 1-D mantle conductivity (Kuvshinov & Olsen 2006) and surface conductance map (Everett et al. 2003)

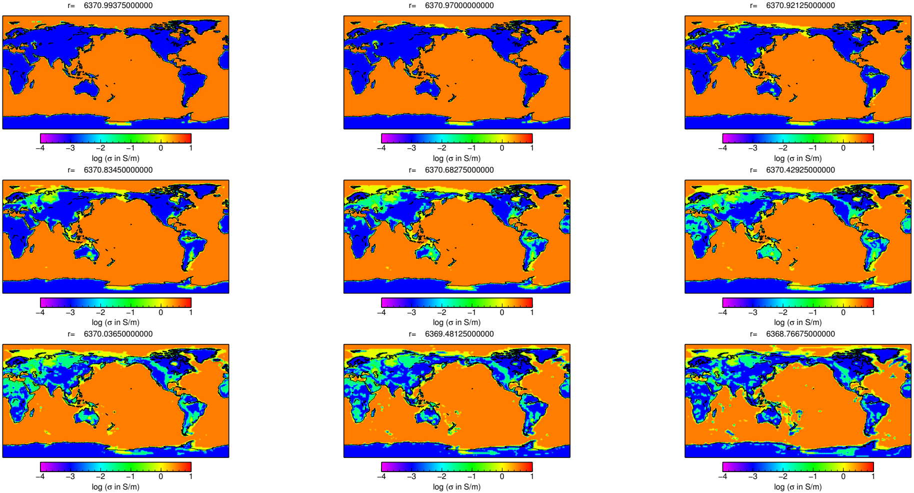
Magnetic field generated by the LSG model

Results



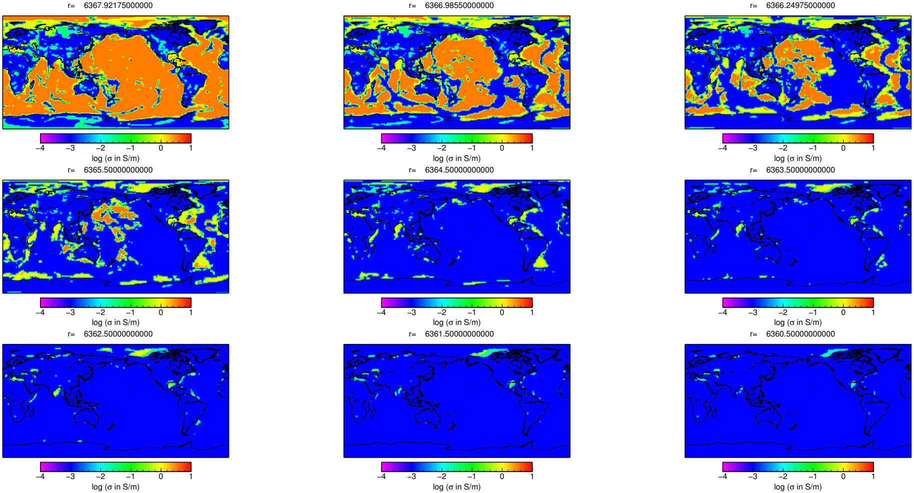
Magnetic field generated by the LSG model

Sensitivity to conductivity model



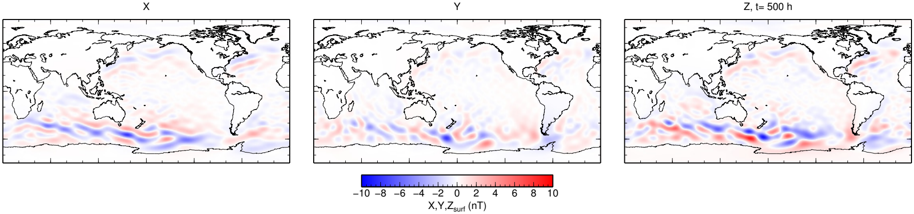
Magnetic field generated by the LSG model

Sensitivity to conductivity model

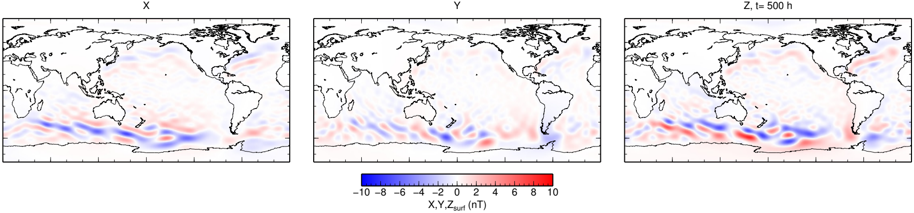


Magnetic field generated by the LSG model

Sensitivity to conductivity model



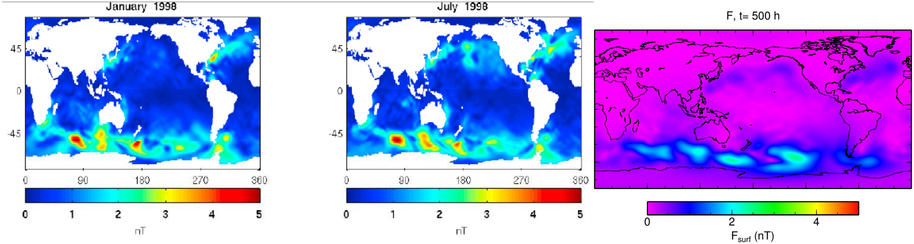
2-D surface conductance



3-D near-surface conductivity

Magnetic field generated by the LSG model

Sensitivity to conductivity model



Amplitude of induced mag. field at 430 km: Left: January 1998 Middle: July 1998 (Glatzman & Golubev 2005)
Right February 2013

WP3000 Status

Tasks completed from PM3 to PM4

- ✓ benchmarks
- ✓ Gaussian vs. bilinear interpolation
- ✓ resolution tests
- ✓ DEBOT parameter sensitivity study
- ✓ IGRF11 vs CHAOS-5
- ✓ initial LSG runs

WP3000 Status

Start: KO

End: MTR

Responsible: JV

Inputs: BTOF, BCOF, MC1D,MC3D,IGRF,MFL2

Outputs: MSBT, MSBC

Activities:

- ✓ Incorporation of motion-generated source term into EM induction code.
- ✓ Testing against analytical solutions.
- ✓ Analysis of effects of spatial resolution.
- ✓ Quantification of effects of choice of magnetic field model and mantle conductivity model.
- ✓ Prediction of magnetic signatures of barotropic models
- ✓ Prediction of magnetic signatures of baroclinic models